Scattering by a small object close to an interface. I. Exact-image theory formulation

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Exact-image theory is applied to the problem of electromagnetic wave scattering from a small dielectric object above an interface separating two isotropic and homogeneous media. The object is assumed to be electrically small and far enough from the interface so that its internal field can be assumed to be uniform. The approach is applicable to any scatter that can be represented by an electric dipole.

INTRODUCTION

The solution for electromagnetic scattering by dielectric objects isolated in space and illuminated by a plane wave is well established. For example, the Mie solution has provided the basis for understanding the light-scattering properties of isolated spherical dielectric objects.[1–3] More recently, numerical techniques have been developed for obtaining scattering results for nonspherical objects, such as spheroids[4] and more general shapes.[5,6]

Almost all the available scattering solutions require that the scattering object be isolated in space; i.e., no other objects or surfaces should be in close proximity. It is becoming increasingly important that one be able to calculate the scattering by objects on or near a dielectric surface.

Two similar solutions for the scattering by a dielectric sphere on or near a dielectric surface have recently been published.[7,8] However, a general solution suitable for problems involving nonspherical objects is not yet available.

The image method is an attractive approach to solving scattering problems involving objects over an interface separating two half-spaces, because it permits half-space solutions based on existing solutions for isolated objects. However, the simple procedure of replacing the material half-space by a suitable image of the object, while possible for objects over a perfectly conducting half-space, is not possible for the dielectric half-space problem. A more general approach, the Sommerfeld solution,[9] can be applied to the dielectric half-space problem. However, this solution depends on the evaluation of the Sommerfeld integrals,[10] a task that is possible only for a limited class of problems.

The exact-image theory (EIT) was introduced in 1984[11–13] as a conceptually and computationally simple means of accounting for an interface of two media in electromagnetic problems, otherwise treated with Sommerfeld integrals.[14] EIT has since been applied, for example, to computing impedances of antennas above the ground[15] and extended to more complicated problems such as multilayered media.[16] For the most recent account of the theory, see Ref. 17.

EIT can potentially be applied to a broad class of problems involving scattering objects above a dielectric half-space. In the present paper EIT is used for deriving a generalized Green function that takes into account the presence of a nearby interface.

In a companion paper[18] the present theory will be applied to the analysis of scattering from spherical objects above an interface with discussion on polarization effects.

THEORY

Consider a planar interface $z = 0$ separating two media: free space, where $z > 0$ with the parameters ϵ_0, μ_0, and ground, with the parameters $\epsilon\epsilon_0$, $\mu\mu_0$. The problem could easily be generalized for any combination of parameters on either side of the interface.

The incident field is a plane wave in air with the electric field

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}_0 \exp(-j\mathbf{k}\cdot\mathbf{r}), \tag{1}$$

where the harmonic time dependence $\exp(j\omega t)$ is assumed. The effect of the ground is to produce the reflected field

$$\mathbf{E}^r(\mathbf{r}) = \mathcal{R}\cdot\mathbf{E}_0 \exp(-j\mathbf{k}_c\mathbf{r}). \tag{2}$$

The subindex c denotes the mirror-image operation through the mirror dyadic

$$\mathcal{C} = \mathcal{I} - 2\mathbf{u}_z\mathbf{u}_z, \tag{3}$$

which changes the sign of the z component of the vector in question:

$$\mathbf{k}_c = \mathcal{C}\cdot\mathbf{k} = \mathbf{k} - 2\mathbf{u}_z(\mathbf{u}_z\cdot\mathbf{k}). \tag{4}$$

The reflection dyadic $\mathcal{R}$ can be written in terms of its TE and TM parts for the plane wave as follows:

$$\mathcal{R} = \mathcal{C}\cdot\left[R^{\mathrm{TE}}\frac{\mathbf{u}_z\mathbf{u}_z \overset{\times}{\times} \mathbf{kk}}{(\mathbf{u}_z\times\mathbf{k})^2} + R^{\mathrm{TE}}\frac{\mathbf{kk}\overset{\times}{\times}(\mathbf{u}_z\mathbf{u}_z\overset{\times}{\times}\mathbf{kk})}{k^2(\mathbf{u}_z\times\mathbf{k})^2}\right], \tag{5}$$

where the double cross product $\mathbf{u}_z\mathbf{u}_z \overset{\times}{\times} \mathbf{kk} = (\mathbf{u}_z\times\mathbf{k})(\mathbf{u}_z\times\mathbf{k})$.

R^{TE} and R^{TM} are the (scalar) Fresnel coefficients for an incident wave with the electric field polarized either perpendicular (TE) or parallel (TM) to the plane of incidence (the plane containing $\mathbf{k}$ and $\mathbf{u}_z$) as shown in Fig. 1:

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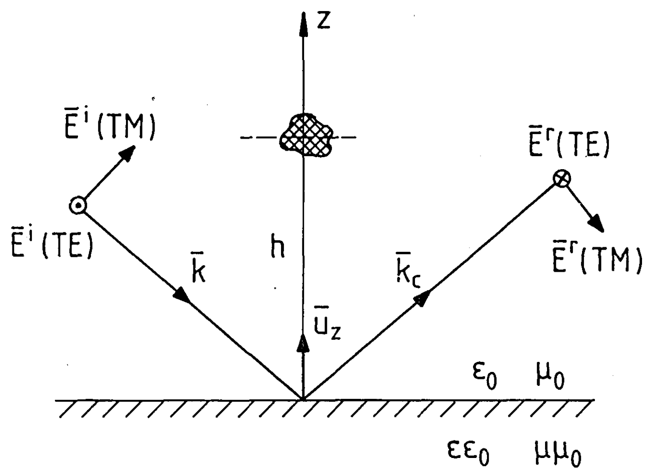

Fig. 1. Geometry of the problem: a scatterer above an interface separating two half-spaces of different media with the definition of TE and TM field vectors.

$$R^{\mathrm{TE}} = \frac{\mu\beta - \beta_1}{\mu\beta + \beta_1}, \tag{6}$$

$$R^{\mathrm{TM}} = -\frac{\epsilon\beta - \beta_1}{\epsilon\beta + \beta_1}, \tag{7}$$

where

$$\beta = [k^2 - (\mathbf{u}_z \times \mathbf{k})^2]^{1/2},$$

$$\beta_1 = [\epsilon\mu k^2 - (\mathbf{u}_z \times \mathbf{k})^2]^{1/2}, \tag{8}$$

$$k^2 = \omega^2\mu_0\epsilon_0, \tag{9}$$

with ω being the angular frequency. $\mathcal{R}$ defines the reflected electric field for any combination of TE or TM incident waves.

The scatterer is a dielectric object located at position $\mathbf{r} = \mathbf{u}_z h$. It is assumed to be sufficiently small that its scattered field can be approximated by the field arising from a dipole with the dipole moment

$$\mathbf{p} = \boldsymbol{\alpha} \cdot \mathbf{E}^t, \tag{10}$$

where $\boldsymbol{\alpha}$, a dyadic, is the tensor dielectric polarizability. Explicit expressions for $\boldsymbol{\alpha}$ are available for isotropic and anisotropic spheres and ellipsoids.[3] $\mathbf{E}^t$, the total field at the location of the scattering object, is the sum of the electric field of the original plane wave, its reflection from the interface, and the net reflected field due to the polarized dielectric object itself. The last component is unknown, and the EIT can be applied to its computation.

The goal is to solve for the unknown dipole moment vector $\mathbf{p}$. The corresponding current density vector can be written as

$$\mathbf{J}(\mathbf{r}) = j\omega\mathbf{p}\delta(\mathbf{r} - \mathbf{u}_z h). \tag{11}$$

The image current due to the interface can be written from the EIT[17] as

$$\mathbf{J}_i(\mathbf{r}, \zeta) = \left[f^{\mathrm{TM}}(\zeta)\mathcal{I} + \frac{1}{k^2} g(\zeta)\mathbf{u}_z\mathbf{u}_z \overset{\times}{\times} \nabla\nabla\right] \cdot \mathbf{J}_c(\mathbf{r}), \tag{12}$$

where both $f^{\mathrm{TM}}(\zeta)$ and $g(\zeta)$ are functions of ϵ and μ, the electrical parameters of the half-space[19,20]:

$$f^{\mathrm{TE}}(\zeta) = jBf(\mu, p) + \frac{\mu - 1}{\mu + 1}\delta_+(p), \tag{13}$$

$$f^{\mathrm{TM}}(\zeta) = -jBf(\epsilon, p) - \frac{\epsilon - 1}{\epsilon + 1}\delta_+(p), \tag{14}$$

$$g(\zeta) = -\frac{\mu^2 - 1}{\mu(\mu - \epsilon)} jBf(\mu, p) - \frac{\epsilon^2 - 1}{\epsilon(\epsilon, \mu)} jBf(\epsilon, p), \tag{15}$$

$$f(\gamma, p) = -\frac{8\gamma}{\gamma^2 - 1}\sum_{n=1}^{\infty} n\left(\frac{\gamma - 1}{\gamma + 1}\right)^n \frac{J_{2n}(p)}{p} U_+(p), \tag{16}$$

with

$$p = jB\zeta \tag{17}$$

and $B = k(\mu\epsilon - 1)^{1/2}$. Here, $\delta_+(p)$ is a delta function with the singularity point at $\zeta = 0_+$ such that

$$\delta_+(0) = 0, \tag{18}$$

and $U_+(p)$ is the corresponding unit step function. In the Bessel functions $J_{2n}(p)$, the complex variable ζ is chosen so that the argument $p = jB\zeta$ is real, thereby ensuring that the Bessel functions converge.

The mirror image of the current dipole is

$$\mathbf{J}_c(\mathbf{r}) = \mathcal{C} \cdot \mathbf{J}(\mathcal{C} \cdot \mathbf{r}) = (\mathcal{I} - 2\mathbf{u}_z\mathbf{u}_z) \cdot j\omega\mathbf{p}\delta(\mathbf{r} + \mathbf{u}_z h). \tag{19}$$

The total electric field at the object, needed for the determination of the dipole moment, can be written as

$$\mathbf{E}^t(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^r(\mathbf{r}) + \mathbf{E}^s(\mathbf{r}), \tag{20}$$

where the field $\mathbf{E}^s$ from the image of the dipole is

$$\mathbf{E}^s(\mathbf{r}) = -j\omega\mu_0 \int_V \int_C \mathcal{G}(\mathbf{r} - \mathbf{r}' + \mathbf{u}_z\zeta) \cdot \mathbf{J}_i(\mathbf{r}', \zeta)\mathrm{d}V'\mathrm{d}\zeta, \tag{21}$$

where V denotes the integration volume of the mirror image and C is a line in the complex ζ plane to be defined later, ranging from the origin to infinity in such a way that the image function integrals converge. $\mathcal{G}$ denotes the free-space Green dyadic

$$\mathcal{G}(\mathbf{r}) = \left(\mathcal{I} + \frac{1}{k^2}\nabla\nabla\right)G(\mathbf{r}), \tag{22}$$

$$G(\mathbf{r}) = \frac{\exp[-jkD(\mathbf{r})]}{4\pi D(\mathbf{r})}, \qquad D(\mathbf{r}) = \sqrt{\mathbf{r} \cdot \mathbf{r}}. \tag{23}$$

Since the image source point does not coincide with the field point at the object, the order of differentiation and integration can be interchanged and the integration over the ζ variable can be carried out. Thus it is possible to define two new Green functions

$$K^{\mathrm{TM}}(\mathbf{r}) = \int_C G(\mathbf{r} + \mathbf{u}_z\zeta)f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta, \tag{24}$$

$$L(\mathbf{r}) = \int_C G(\mathbf{r} + \mathbf{u}_z\zeta)g(\zeta)\mathrm{d}\zeta \tag{25}$$

and a corresponding Green dyadic

$$\mathscr{K}(\mathbf{r}) = \left(\mathscr{I} + \frac{1}{k^2}\nabla\nabla\right)K^{\mathrm{TM}}(\mathbf{r}) + \frac{1}{k^2}(\mathbf{u}_z\mathbf{u}_z \overset{\times}{\times} \nabla\nabla)L(\mathbf{r}). \quad (26)$$

The field from the image source can now be expressed more concisely as

$$\mathbf{E}^s(\mathbf{r}) = -j\omega\mu_0 \int_V \mathscr{K}(\mathbf{r}-\mathbf{r}')\cdot\mathbf{J}_c(\mathbf{r}')\mathrm{d}V'. \quad (27)$$

Inserting the proper mirror-image current function from Eq. (19), we have

$$\mathbf{E}^s(\mathbf{r}) = \omega^2\mu_0\mathscr{K}(\mathbf{r}+\mathbf{u}_z h)\cdot(\mathscr{I} - 2\mathbf{u}_z\mathbf{u}_z)\cdot\mathbf{p}, \quad (28)$$

which contains the unknown dipole moment vector $\mathbf{p}$, for which a final equation can now be formed. In fact, combining Eqs. (10), (20), and (28) for the point $\mathbf{r} = \mathbf{u}_z h$, we arrive at an algebraic equation for the moment vector:

$$\mathbf{p} = \alpha\cdot\mathbf{E}^t(\mathbf{u}_z h) = \alpha\cdot[\mathbf{E}^i(\mathbf{u}_z h) + \mathbf{E}^r(\mathbf{u}_z h)] + \omega^2\mu_0\alpha\cdot\mathscr{K}(2\mathbf{u}_z h)\cdot\mathscr{C}\cdot\mathbf{p}. \quad (29)$$

This has the solution

$$\mathbf{p} = [\mathscr{I} - \omega^2\mu_0\alpha\cdot\mathscr{K}(2\mathbf{u}_z h)\cdot\mathscr{C}]^{-1}\cdot\alpha\cdot[\mathbf{E}^i(\mathbf{u}_z h) + \mathbf{E}^r(\mathbf{u}_z h)]. \quad (30)$$

In numerical integrations, it is important to choose the integration path in the complex ζ plane in such a way that the Bessel functions of the image currents converge as mentioned above; i.e., their argument $p = jB\zeta$ has to be real. This entails that ζ has to proceed to infinity along a path such that

$$\arg\{\zeta\} = -\pi/2 - \arg\{\sqrt{\epsilon\mu - 1}\}. \quad (31)$$

For the integrations in the complex plane, see Ref. 20.

EVALUATION OF THE GREEN FUNCTION

In evaluating the total dipole moment of the scatterer [Eq. (30)], the essential step is to enumerate the Green function $\mathscr{K}(\mathbf{r})$ in Eq. (26) for $\mathbf{r} = 2\mathbf{u}_z h$.

Making use of the fact that this dyadic is not dependent on the transverse directions and will be evaluated at the z axis and the fact that, with the origin excluded, the double gradient of a function with a Green-function-type r dependence [see Eqs. (23)] is

$$\nabla\nabla f(r) = \left[-k^2\mathbf{u}_r\mathbf{u}_r - (\mathscr{I} - 3\mathbf{u}_r\mathbf{u}_r)\frac{1+jkr}{r^2}\right]f(r), \quad (32)$$

we can write the $\mathscr{K}$ dyadic in the form

$$\mathscr{K}(z\mathbf{u}_z) = \mathscr{I}_t\left(K^{\mathrm{TM}}(z) + \int_C G(z+\zeta)\left\{\frac{1}{[jk(z+\zeta)]^2} + \frac{1}{jk(z+\zeta)}\right\}[f^{\mathrm{TM}}(\zeta) + g(\zeta)]\mathrm{d}\zeta\right) - 2\mathbf{u}_z\mathbf{u}_z\left(\int_C G(z+\zeta)\left\{\frac{1}{[jk(z+\zeta)]^2} + \frac{1}{jk(z+\zeta)}\right\}f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta\right), \quad (33)$$

where $\mathscr{I}_t = \mathscr{I} - \mathbf{u}_z\mathbf{u}_z$ denotes the two-dimensional unit dyadic.

For the nonmagnetic-ground case ($\mu = 1$), Eq. (33) becomes, after transformation to a real integration parameter p,

$$\mathscr{K}(z\mathbf{u}_z) = \mathscr{I}_t\left(-\frac{\epsilon-1}{\epsilon+1}G(z)\left[1 + \frac{1+jkz}{(jkz)^2}\right] + \frac{8\epsilon}{\epsilon^2-1}\int_0^\infty G(z - jp/B)\left\{1 + \frac{(2\epsilon+1)}{\epsilon}\cdot\frac{[1+jk(z-jp/B)]}{[jk(z-jp/B)]^2}\right\}\sum_{n=0}^{\infty} n\left(\frac{\epsilon-1}{\epsilon+1}\right)^n \frac{J_{2n}(p)}{p}\mathrm{d}p\right) - 2\mathbf{u}_z\mathbf{u}_z\left\{-\frac{\epsilon-1}{\epsilon+1}G(z)\frac{1+jkz}{(jkz)^2} + \frac{8\epsilon}{\epsilon^2-1}\int_0^\infty G(z-jp/B) \times \frac{1+jk(z-jp/B)}{[jk(z-jp/B)]^2}\sum_{n=0}^{\infty} n\left(\frac{\epsilon-1}{\epsilon+1}\right)^n\frac{J_{2n}(p)}{p}\mathrm{d}p\right\}. \quad (34)$$

The components of this Green dyadic are $K_t(z)$ and $K_z(z)$, and, for the enumeration of the total dipole moment, the dyadic must be evaluated for the argument $z = 2h$:

$$\mathscr{K}(z\mathbf{u}_z) = \mathscr{I}_t K_t(z) + \mathbf{u}_z\mathbf{u}_z K_z(z). \quad (35)$$

In Figs. 2–5 the Green function components are calculated for the relative permittivity values $\epsilon = 1.7$ and $\epsilon = 2.4$ of the ground as functions of kh.

ASYMPTOTIC TESTS

Consider three special cases for the scattering problem above a nonmagnetic ground ($\mu = 1$). First, let $h \to \infty$. Here, the $\mathscr{K}$ dyadic in Eq. (33) can be approximated by its

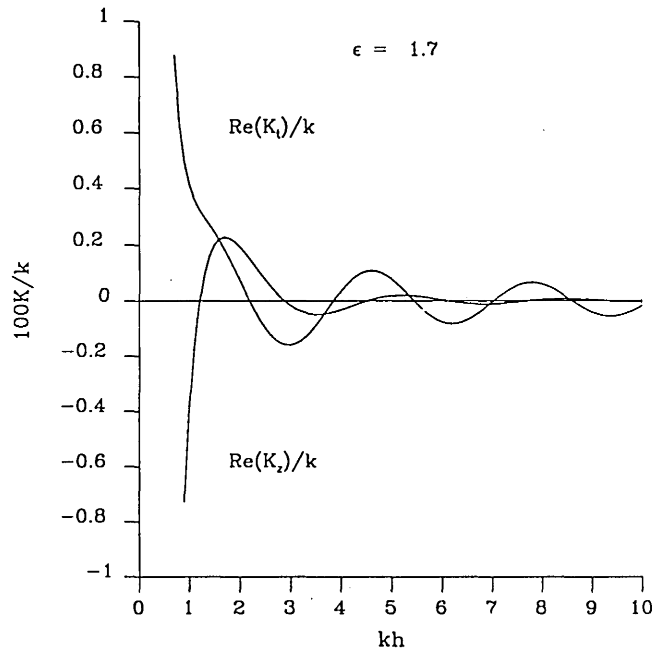

Fig. 2. Real parts of the Green dyadic $\mathscr{K}(z\mathbf{u}_z)$ components of Eq. (34) as functions of the normalized scatterer height kh for ground parameters $\mu = 1$ and $\epsilon = 1.7$. Note that for large kh values K_z decays faster than K_t. Also, the asymptotic dependence of the Green dyadic is according to Eq. (36).

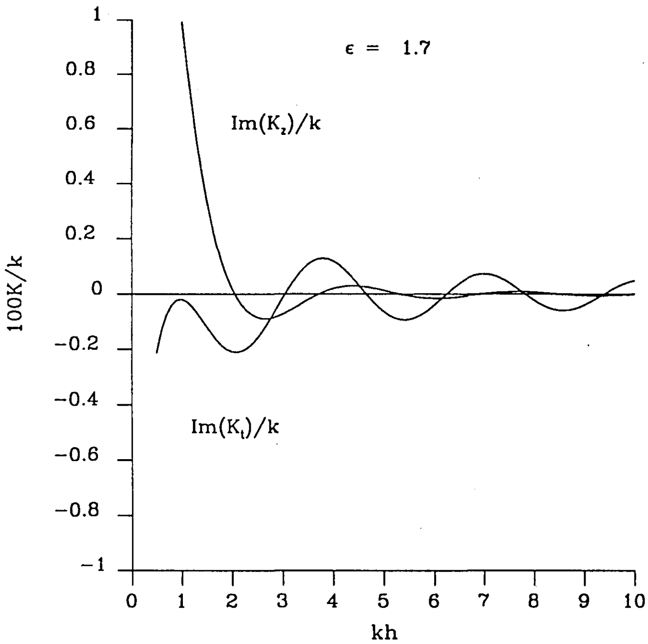

Fig. 3. Same as Fig. 2 for the imaginary part.

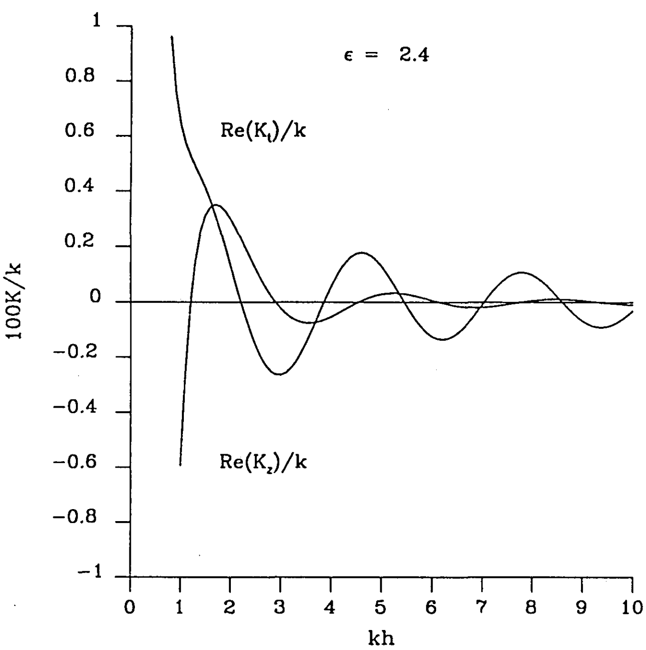

Fig. 4. Same as Fig. 2 for ϵ =2.4.

first term because the other terms from the ζ integration yield terms of the order of h^{-2} and h^{-3}:

$$K^{\mathrm{TM}}(2h\mathbf{u}_z) \simeq \frac{\exp(-jk2h)}{4\pi 2h}\int_C \exp(-jk\zeta)f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta$$

$$= G(2h)R^{\mathrm{TM}}(k), \qquad (36)$$

and both integrations over the image functions in the transversal and normal components of the dyadic are of the order of h^{-2}. Therefore, to the leading order, only the transversal component [the term in expression (36)] remains.

The last step in expression (36) is due to the definition of the image function f^{TM}; see, for example, Ref. 13. The whole $\mathcal{K}$ dyadic is therefore

$$\mathcal{K}(2h\mathbf{u}_z) \simeq G(2h)R^{\mathrm{TM}}(k)\mathcal{I}_t, \qquad (37)$$

which is what one would anticipate. Note that for this case of interaction through normally incident wave components, the reflection coeficient is

$$R^{\mathrm{TM}}(k) = R^{\mathrm{TE}}(k) = \frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}. \qquad (38)$$

For example, the total TE dipole moment can be written from Eq. (30). Because $G(2h)$ is small and because the mirror-image operator $\mathcal{C}$ does not affect a transverse dyadic, the dipole moment in the case of a scalar scatterer polarizability ($\boldsymbol{\alpha} = \alpha\mathcal{I}$) is

$$\mathbf{p} = \alpha[1 - \omega^2\mu_0\alpha K^{\mathrm{TM}}(2h\mathbf{u}_z)]^{-1}(\mathbf{E}^i + \mathbf{E}^r)$$

$$\simeq \alpha[1 + \omega^2\mu_0\alpha R^{\mathrm{TM}}G(2h)](\mathbf{E}^i + \mathbf{E}^r)$$

$$= \alpha(\mathbf{E}^i + \mathbf{E}^r) + \omega^2\mu_0\alpha R^{\mathrm{TM}}G(2h)\alpha(\mathbf{E}^i + \mathbf{E}^r), \qquad (39)$$

which can be interpreted as the sum of the dipole moment in free space and that due to the first-order interaction [the image dipole being $\alpha(\mathbf{E}^i + \mathbf{E}^r)R^{\mathrm{TM}}$].

The case $\epsilon = 1$, in which the interface becomes transparent, simplifies the image functions of Eqs. (14) and (15):

$$f^{\mathrm{TM}}(\zeta) = \lim_{\epsilon\to 1}\left[-jBf(\epsilon, jB\zeta) - \frac{\epsilon-1}{\epsilon+1}\delta_+(\zeta)\right]$$

$$= \lim_{\epsilon\to 1}\frac{-\epsilon B^2}{(\epsilon+1)^2}\zeta = 0, \qquad (40)$$

$$g(\zeta) = \lim_{\epsilon\to 1}\left[-\frac{\epsilon+1}{\epsilon}jBf(\epsilon, jB\zeta)\right] = \lim_{\epsilon\to 1}\frac{-B^2}{\epsilon+1}\zeta = 0. \qquad (41)$$

Interpreted in words, the integration over ζ shrinks to a point because $B = 0$, which multiplies ζ in the argument of

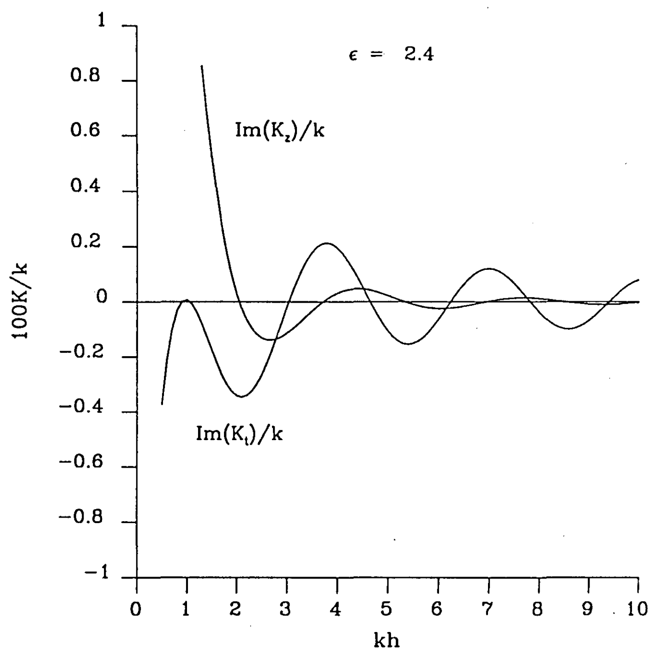

Fig. 5. Same as Fig. 3 for ϵ = 2.4.

the Bessel function, and, at that very point, the image functions are 0, giving rise to no scattered field over the whole integration path.

The case $\epsilon = \infty$ also forces B to infinity, and the integral in $K_{\mathrm{TM}}(t)$ gives

$$K^{\mathrm{TM}}(qh\mathbf{u}_z) = \frac{\exp(-jk2h)}{4\pi 2h}\int_C \left[\frac{\exp(-jk\zeta)}{1+(\zeta/2h)}\right] f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta. \quad (42)$$

In performing this integral, the transformation $\zeta = -jp/B$ has to be made. The bracketed term in the integrand tends to 1:

$$\int_C \frac{\exp(-jk\zeta)}{1+(\zeta/2h)} f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta = \int_C f^{\mathrm{TM}}(\zeta)\mathrm{d}\zeta = R^{\mathrm{TM}}(0) = -1. \quad (43)$$

Hence this limit case is correct because only the mirror image is left from the Green function corresponding to the case of ideally conducting plane.

One could also wonder what happens as the height h decreases to 0. A superficial look at Eq. (30) and Figs. 2–5 might give the impression that there is no scattered field as the scatterer is located at the ground, because for $h = 0$ the Green dyadic $\mathcal{K}$ goes to infinity, forcing the total dipole moment to 0. However, a spherical scatterer cannot be pushed closer to the boundary than its radius, and, if one wishes to decrease h even from that value, the radius has to be decreased, and then also the polarizability (being proportional to the volume of the scatterer) decreases toward 0, keeping the product $\boldsymbol{\alpha} \cdot \mathcal{K}$ finite.

CONCLUSIONS

The result of the theory presented in this paper culminates in the Green dyadic [Eq. (34)], which, with Eq. (30) permits one to determine the fields scattered by a polarizable particle in the presence of an interface. The Green dyadic is not a multiple of the unit dyadic, meaning that through Eq. (30) scalar polarizability of a scatterer is transformed to dyadic polarizability that takes the boundary into account. From this follows, for example, that a sphere close to an interface is equivalent to a uniaxial ellipsoid in free space.

The result is an approximation with the assumption that the scatterer with the nearby ground can be replaced by an electric dipole. This approximation is valid for small scatterers for which the internal field is close to constant. It is clear that as the scatterer is not too close to the interface, the approximation holds. With decreasing kh, the assumption starts to fail, and at some point it cannot be used at all. This occurs when the higher multipoles are strong compared with the electric dipole. To take these multipoles into account, we can make the first improvement by taking two parallel vector equations, one for the electric and the other for the magnetic dipole, and solving them. A more general approach would be to derive an integral equation for the polarization current and solve that. However, the range of validity (in terms of kh) of the electric-dipole approximation presented in this paper cannot be determined until a more accurate method of solving the problem is discovered. The corresponding example of thin horizontal cylindrical dipoles above ground[15] did not fail until reaching heights of the order of the diameter of the dipole.

In a companion paper[18] this theory is applied to the study of the scattering properties of spherical objects in the presence of the ground, with special emphasis on the effect of backscattering enhancement.

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