Electromagnetic Scattering I (53919, 5 cr)
Exercise 2

Ex tempore exercises 1-2 (answers discussed during the lectures):

1. Consider two small non-interacting spherical particles of radius $a << \lambda$, where λ is the wavelength of the incident plane wave (size parameter $x = 2\pi a/\lambda$). One particle is set to the origin and the location of the other particle is denoted by a vector $\mathbf{d}$. Study the interference in first-order scattering when the internal fields are assumed to coincide with the incident field at the particle locations.

2. Calculate the form factor for a cubic particle.

Home exercises 1-2 (return your answers by Tuesday, 4 pm, Sept. 20, 2016):

1. A plane wave is scattered by two small interacting spherical particles of radius $a << \lambda$, where λ is the wavelength of the incident plane wave (size parameter $x = 2\pi a/\lambda$). One particle is set to the origin and the location of the other particle is denoted by a vector $\mathbf{d}$. In the dipole approximation, the internal fields of the particles $\mathbf{E}_1$ and $\mathbf{E}_2$ are related through

$$\begin{aligned}\mathbf{E}_1 &= \mathbf{E}_{i1} + \beta\bar{\mathbf{T}}(u,v)\cdot\mathbf{E}_2 \\ \mathbf{E}_2 &= \mathbf{E}_{i2} + \beta\bar{\mathbf{T}}(u,v)\cdot\mathbf{E}_1,\end{aligned}$$

where $\mathbf{E}_{i1}$ and $\mathbf{E}_{i2}$ are the incident fields at the locations of the particles, and the polarizability (m is the refractive index)

$$\beta = x^3\frac{m^2-1}{m^2+2}.$$

The transformation $\bar{\mathbf{T}}$ denotes the interaction between the particles:

$$\begin{aligned}\bar{\mathbf{T}}(u,v) &= u\bar{\mathbf{I}} + v\mathbf{dd}/d^2 \\ u &= \mathrm{e}^{i\rho}(\rho^2 + i\rho - 1)/\rho^3 \\ v &= \mathrm{e}^{i\rho}(-\rho^2 - i3\rho + 3)/\rho^3,\ \ \rho = kd.\end{aligned}$$

Solve the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$. (See Muinonen 1990, PhD thesis.)

Note the following rules for an operator $\mathbf{ab}$:
$(\mathbf{ab})\cdot\mathbf{c} = \mathbf{a}(\mathbf{b}\cdot\mathbf{c})$
$\mathbf{c}\cdot(\mathbf{ab}) = (\mathbf{c}\cdot\mathbf{a})\mathbf{b}$
The unit operator $\bar{\mathbf{I}}$ has no effect on the operator $\mathbf{ab}$ or the vector $\mathbf{c}$.

(12 points)

2. Derive the scattering matrix elements for the particle system in Exercise 2.1 in a fixed orientation. What is the scattering matrix for the specific configurations where the vector $\mathbf{d}$ is

(i) perpendicular to the scattering plane;
(ii) within the scattering plane but perpendicular to the wave vector of the incident field.

(18 points)