Electromagnetic Scattering I (53919, 5 cr)
Exercise 1

Ex tempore exercises 1-2 (answers discussed during the lectures):

1. Starting from the Maxwell equations, derive the vector Helmholtz equations for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$:

$$\begin{aligned} \nabla^2 \mathbf{E} + k^2 \mathbf{E} &= 0, \\ \nabla^2 \mathbf{H} + k^2 \mathbf{H} &= 0. \end{aligned}$$

2. Starting from the Poynting vector for the scattered field and the asymptotic far-field form of the field, the scattered power per unit area far away from the scattering particle.

Home exercises 1-5 (return your answers by Monday noon, Sept. 12, 2016):

1. Derive the following relation between the Stokes parameters $\mathbf{I} = (I, Q, U, V)^T$ and the ellipsometric parameters

$$\begin{aligned} I &= c^2 \\ Q &= c^2 \cos 2\eta \cos 2\gamma \\ U &= c^2 \cos 2\eta \sin 2\gamma \\ V &= c^2 \sin 2\eta, \end{aligned}$$

where $c^2 = a^2 + b^2$ (a and b being the semimajor and semiminor axis, respectively), γ the clockwise angle between $\mathbf{e}_\parallel$ (Bohren-Huffman, p. 50) and major axis, and $|\tan \eta| = b/a$ (ellipticity). Hint: the form of the plane wave remains invariant when expressed in differing orthogonal coordinate systems. (6 points)

2. Draw the vibration ellipses for the Stokes parameters $\mathbf{I} = (3, 1, 2, -2)^T$ and $\mathbf{I} = (25, 0, 24, 7)^T$. (6 points)

3. An electromagnetic plane wave propagating in a nonabsorbing medium is normally incident on an infinite slab between $z = 0$ and $z = h$ with a refractive index $m_1 = n_1 + ik_1$. Derive the expressions for the slab reflectance and transmittance. (12 points)

4. Derive expressions for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ in the electric dipole approximation. Vector potential is

$$\mathbf{A}(\mathbf{r}) = \frac{-ikc\mu_0}{4\pi} \, \mathbf{p} \, \frac{e^{ikr}}{r},$$

where $\mathbf{p}$ is the electric dipole moment. (Jackson 9.2; 12 points)

5. Based on the results of Problem 4 in Exercise 1, derive the scattered electromagnetic field in the radiation zone (far zone), and all the elements of the Rayleigh scattering matrix. The relation between the scattering matrix and amplitude scattering matrix elements is presented, e.g., in Bohren & Huffman. (12 points)