1 (lecture 14)

In the PM-method (point matching), the boundary conditions of the electromagnetic fields are required in a finite number of points on the surface of the particle. In the original method, there were as many points as unknown coefficients in the vector spherical harmonics expansion. It was concluded that the method was numerically instable. There is, however, nothing that prevents us from expanding the number of points and computing the coefficients using the least-squares method. This version of the method has been noticed to be stable and is one of the most popular numerical methods. The regime of application can be improved by expanding the fields with a number of suitably chosen origins within the particle. PM is promising also for scattering by Gaussian particles. It is intriguing to ponder whether "an educated guess" can help speed up the solution of the coefficients.

The integral-equation methods are divided into a wide spectrum of different methods. In the VIEM method (volume-integral-equation), one considers the integral equation

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_i(\mathbf{r}) + k^2 \int_V d^3\mathbf{r}' [\mathbf{1} + \frac{1}{k^2}\nabla\nabla] \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \cdot [m^2(\mathbf{r}') - 1]\mathbf{E}(\mathbf{r}').$$

By discretizing the integral on the right-hand side, one obtains a group of linear equations for the field values at the discretization points within the volume of the particle. Solving the equations results in the field inside the particle. Typically, again, 10-20 discretization points are required as per wavelength so that, after a straightforward calculation, it is clear that a group of equations with thousands of unknowns easily follows. In practice with current computers, up to 200 million unknowns can be treated (as of December 12, 2008). Various versions of the VIEM method have been successfully applied to Gaussian-particle scattering (foremost DDA, discrete-dipole approximation).

In the case of VIEM, the matrix of the group of linear equations is full, which makes the solution more difficult. When the internal field has been solved for, the same integral relation gives the scattered field outside the particle via straightforward integration (subtracting the original field).

DDA (discrete-dipole approximation) is a certain version of solution methods for the integral equation. DDA can be visualized in the following: the particle can be thought to be composed of dipole scatterers interacting with each other. In practice, the VIEM methods differ from one another in how they treat the singular self-term inside the integral, which is essential for the accuracy of the method.

The surface-integral-equation methods (SIEM) make use of two-dimensional integral equations that seem like a reasonable starting point, in particular, for homogeneous particles. However, the SIEM-methods are less stable than the VIEM-methods and usually require additional regularization.

The integral equation shown above in connection to the VIEM-method is Fredholm-type and the kernel has a singularity at $\mathbf{r} = \mathbf{r}'$. Via Fourier-transformation, handling of the singularity can be improved and the integral equation can be solved numerically in the wavenumber (or frequency) space. Surprisingly, the disadvantage of the method is the considerable analytical work needed for each different particle. These so-called FIEM-methods have not been very popular.

In the TMM method (transition matrix method), the analysis proceeds with the help of vector spherical harmonics functions and the word "transition" refers to the linear matrix relation between the original field and the scattered field. Compared to the direct vector spherical harmonics treatment of the boundary conditions, TMM has the advantage that a linear relation is obtained purely between the internal and original fields, reducing the number of unknowns in the group of linear equations. After solving the group of equations, the scattered is computed from the vector Kirchhoff integral relation. The TMM method is an efficient method, in particular, for axially symmetric particles and useful results have been obtained, e.g., for spheroids to compare with the implications of the SVM method. However, TMM suffers from unpredictable convergence and instability problems and have not yet been extensively applied to scattering by Gaussian particles. As a tool the actual T-matrix is quite useful and, for a single particle, needs to be computed only once (independently of the orientation). Recently, an analytical version of the T-matrix method has been developed—this version is highly promising for studying scattering by Gaussian random particles.

In the superposition method for spheres and spheroids, scattering by particle clusters is computed using the translation and addition rules of vector spherical harmonics functions. The field scattered by the cluster is expressed as a superposition of the fields scattered by each constituent particle. The partial fields depend on each other due to the mutual electromagnetic interactions of the constituent particles. The scattering problem again manifests itself in a solution of a group of linear equations. Currently, precise solutions can be computed for clusters with several dozens of constituent particles, when constituent-particle size approaches the wavelength.

2 Applications of electromagnetic scattering

In his book, van de Hulst has presented an excellent review of the applications of light scattering in various fields of science. This is recommended reading bearing in mind, in particular, modern computational methods for nonspherical particles. Bohren and Huffman offer additional material on the applications, as well as Mishchenko et al. Finally, the publications from the meeting series entitled *Electromagnetic and Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications* offer up-to-date information about the advances in light scattering by small particles.