Computational light scattering, fall 2020 (PAP315, 5 cr)
Exercise 5

1. Consider radiative transfer in a spherical medium (ignoring polarization) with radial optical thickness τ composed of Henyey-Greenstein scatterers with the single-scattering albedo $\tilde{\omega}$ and phase function P_{HG} given in Exercise 4. In the case of unidirectional unpolarized incident radiation (e.g., the Sun as the source), compute the angular distribution of scattered radiation as well as the amount of absorbed radiation by writing a Monte Carlo computer program for the radiative transfer problem at hand. As an example, depict the angular distribution for $\tau = 2$, $\tilde{\omega} = 0.9$, and $g = 0.6$ in P_{HG}. (15 points)

2. Consider Monte Carlo radiative transfer within a scattering and absorbing medium in the case of a Mie-type block-diagonal scattering phase matrix with nonzero matrix elements $P_{11} = P_{22}$, $P_{12} = P_{21}$, $P_{33} = P_{44}$, and $P_{34} = -P_{43}$. Show that, in the scattering process, a new scattering direction can be derived by using the marginal probability density

$$p(\theta) \quad = \quad \frac{1}{2} P_{11}(\theta) \tag{1}$$

to obtain the sample $\tilde{\theta}$ and, subsequently, by generating the azimuthal angle $\tilde{\phi}$ from (with $y \in]0,1[$ a uniform random deviate)

$$(2\tilde{\phi} + \gamma) - e \sin(2\tilde{\phi} + \gamma) \quad = \quad 4\pi y + \gamma - e \sin \gamma, \tag{2}$$

that is, Kepler's equation $E - e \sin E = M$ with "eccentric anomaly" $E = 2\tilde{\phi} + \gamma$, "eccentricity" e, and "mean anomaly" $M = 4\pi y + \gamma - e \sin \gamma$. Here

$$\begin{aligned} e \cos \gamma \quad &= \quad -\frac{P_{12}(\tilde{\theta})}{P_{11}(\tilde{\theta})} \frac{Q_1}{I_1}, \\ e \sin \gamma \quad &= \quad -\frac{P_{12}(\tilde{\theta})}{P_{11}(\tilde{\theta})} \frac{U_1}{I_1}, \end{aligned} \tag{3}$$

where $\underline{I}_1 = (I_1, Q_1, U_1, V_1)^T$ denotes the Stokes vector (in an arbitrary reference system) of radiation incident on the scatterer. (9 points)

Literature: K. Muinonen, *Waves in Random Media* **14**, 365, 2004.