Computational light scattering, fall 2020 (PAP315, 5 cr)
Exercise 2

1. Consider two small non-interacting spherical particles of radius $a << \lambda$, where λ is the wavelength of the incident plane wave (size parameter $x = 2\pi a/\lambda$). One particle is set to the origin and the location of the other particle is denoted by a vector $\mathbf{d}$. Study the interference in first-order scattering when the internal fields are assumed to coincide with the incident field at the particle locations.

(6 points)

2-3. A plane wave is scattered by two small interacting spherical particles of radius $a << \lambda$, where λ is the wavelength of the incident plane wave (size parameter $x = 2\pi a/\lambda$). One particle is set to the origin and the location of the other particle is denoted by a vector $\mathbf{d}$. In the dipole approximation, the internal fields of the particles $\mathbf{E}_1$ and $\mathbf{E}_2$ are related through

$$\begin{aligned}\mathbf{E}_1 &= \mathbf{E}_{i1} + \beta\bar{\mathbf{T}}(u,v)\cdot\mathbf{E}_2 \\ \mathbf{E}_2 &= \mathbf{E}_{i2} + \beta\bar{\mathbf{T}}(u,v)\cdot\mathbf{E}_1,\end{aligned}$$

where $\mathbf{E}_{i1}$ and $\mathbf{E}_{i2}$ are the incident fields at the locations of the particles, and the polarizability (m is the refractive index)

$$\beta = x^3\frac{m^2-1}{m^2+2}.$$

The transformation $\bar{\mathbf{T}}$ denotes the interaction between the particles:

$$\begin{aligned}\bar{\mathbf{T}}(u,v) &= u\bar{\mathbf{I}} + v\mathbf{dd}/d^2 \\ u &= \mathrm{e}^{i\rho}(\rho^2 + i\rho - 1)/\rho^3 \\ v &= \mathrm{e}^{i\rho}(-\rho^2 - i3\rho + 3)/\rho^3, \ \ \rho = kd.\end{aligned}$$

Solve the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$. (See Muinonen 1990, PhD thesis.)

Note the following rules for an operator $\mathbf{ab}$:
$(\mathbf{ab})\cdot\mathbf{c} = \mathbf{a}(\mathbf{b}\cdot\mathbf{c})$
$\mathbf{c}\cdot(\mathbf{ab}) = (\mathbf{c}\cdot\mathbf{a})\mathbf{b}$
The unit operator $\bar{\mathbf{I}}$ has no effect on the operator $\mathbf{ab}$ or the vector $\mathbf{c}$.

(12 points)

4. Using a Mie scattering computer code, consider the formation of the rainbow and glory phenomena in the angular scattering by water droplets. Fro the droplets, the refractive index is $m = 1.33$ and the size parameter is $x = ka$, where a is the radius of the spherical particle, $k = 2\pi/\lambda$ is the wave number, and λ is the wavelength. Increase x gradually starting from the Rayleigh regime. At what values of x do the phenomena start to show up? Where do the interference patterns in the angular dependencies derive from? Typically, how does the number of interference features within the scattering angle range $0-180°$ relate to x? In the nature, water droplets follow a size distribution. Carry out size averaging for the angular patterns and show that the rainbow and glory features are enhanced. Explain the rainbow features qualitatively using geometric optics. The glory phenomenon cannot be explained using geometric optics. How would you explain it? Provide one explanation based on the literature.

(6 points)