9th exercises for SIM'2020

Ex. 1

Use inversion method from 8.1.1 and derive a formula for drawing random numbers from exponential distribution $f(y) = \lambda \exp(-\lambda y)$. Create 1,000 numbers from $\exp(3)$ and draw histogram or box-and-whiskers plot.

Ex. 2

a) Create two random data from normal distribution. Use Eq. (8.2) and the fact that if $X \sim \mathcal{N}(0, 1)$ then $Y = \sigma X + \mu \sim \mathcal{N}(\mu, \sigma^2)$. Both data should have n = 1,000, but \boldsymbol{y}_1 with $\mu = 2, \sigma^2 = 2$ and \boldsymbol{y}_2 with $\mu = 4, \sigma^2 = 4$.

b) Compute the difference between medians, $m = \text{median}(y_2) - \text{median}(y_1)$. Use bootstrap to resample from both datasets and compute the (bootstrap) 95 % confidence interval for m.

Ex. 3

Let's invent our own "order test". We have array of integers y = (1, 2, 3, 16, 5, 12, 13, 11, 9, 10, 8, 6, 7, 14, 15, 4, 17, 18, 19, 20). It seems like it has some kind of order, but can it be purely by accident? We could compute e.g. a measure of order as

$$t = \sum_{i=1}^{n-1} I(y_i, y_{i+1}), \text{ where } I(a, b) = \begin{cases} 1, \text{ if } a < b \\ -1, \text{ if } a > b \end{cases}$$

Test using permutation test and the statistics *t* if the array has order or is randomly ordered.