1st exercises for SIM'2020

Ex. 1

a) Show with Venn diagram for events A and B that $A = AB^C \cup AB$
b) Show with Venn diagram for events A and B that $A \cup B = AB^C \cup A^C B \cup AB$
c) Use axiom 1.3 from material and previous items a) and b), and derive the addition rule for two events in Eq. 1.4: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(AB)$.

Ex. 2
Let's assume that $\mathrm{P}(A|B) = \mathrm{P}(A|B^C)$. Show that then $A \perp\!\!\!\perp B$.

Ex. 3
Prove that $\mathrm{E}(aU + b) = a\mathrm{E}(U) + b$ for random variable U and constants a, b. Use Eq. 1.17.

Ex. 4
Compute $\mathrm{E}(Y)$, when distribution for Y is
a) $\mathrm{f}(y) = \frac{1}{2}\exp(-|y|),\ y \in \mathbb{R}$
b) $\mathrm{f}(y) = 8/y^3,\ y > 2$
c) $\mathrm{f}(y) = y\exp(-\frac{1}{2}y^2),\ y > 0$

Ex. 5
There is 5 white and 10 black balls in a bowl. Ball is lifted, color checked, and returned to bowl. This is done 10 times. What is the probability to receive a) at least one white ball, b) five white balls?

Ex. 6
Let U have uniform distribution between $(-1, 1)$, so $\mathrm{f}_U(u) = 1/2$. What is the distribution of transformed variable $V = U^2$?

Ex. 7
Download the datafile `two-variable.dat` and make report (including, e.g., statistics of the two variables (columns), dependence, plots...)