10th exercises for SIM'2019

Ex. 1

Consider Fisher's z-distribution (Eq. (9.3)) with $f(x; 2, 10)$ and evaluate $P(X < 0)$. Use importance sampling for that with Cauchy distribution as the instrumental distribution with $g(x; 0, 1/2)$.

Ex. 2

Minimize the Rosenbrock function $h(x, y; a, b) = (a - x)^2 + b(y - x^2)^2$ using simulated annealing. Start from $(-1, -1)$ with the choice $a = 1, b = 100$. Try to plot the function together with the chain of the values found by the simulated annealing algorithm.

Ex. 3

Again, re-do the example with the independent Metropolis-Hastings and Fisher's z-distribution (Eq. (9.3)) in the lecture material, page 9-7. Use Cauchy distribution as the proposal distribution with $g(x; 0, 1/2)$. Plot the behavior of the estimate for the mean of Fisher's z-distribution against the length of the MCMC chain.

Ex. 4

'Show' numerically that the posterior distribution is Gamma when the data follows exponential distribution and the prior distribution for the intensity parameter λ in exponential distribution is Gamma($a,a/b$) (see Ex. 7.1).

The exponential data is $\boldsymbol{x} = (0.254, 0.360, 0.0372, 0.340, 0.252, 0.105, 0.111, 0.222, 0.162, 0.0307)$ and the hyperparameters are $a = 3, b = 3$. 'Show' the posterior using random walk Metropolis-Hastings, and plotting the histogram of the chain.