

1st exercises for SIM'2019

Ex. 1

Show with Venn diagram for events A and B that

a) $A = AB^C + AB$

b) $A \cup B = AB^C \cup A^C B \cup AB$

c) Use axiom 1.3 from material to items a) (for A and B) and b), and derive the addition rule for two events in Eq. 1.4: $P(A \cup B) = P(A) + P(B) - P(AB)$.

Ex. 2

Let's assume that $P(A|B) = P(A|B^C)$. Show that then $A \perp\!\!\!\perp B$.

Ex. 3

Show that $P(A) \leq 1 - P(A^C \cap B^C) \leq P(A) + P(B)$. Hints: Total probability says that $P(S) = 1$, $A \cup A^C = S$. You can use Venn diagrams for certain steps.

Ex. 4

Prove that $E(aU + b) = aE(U) + b$ for random variable U and constants a, b . Use Eq. 1.17.

Ex. 5

Compute $E(Y)$, when distribution for Y is

a) $f(y) = \frac{1}{2} \exp(-|y|)$, $y \in \mathbb{R}$

b) $f(y) = 8/y^3$, $y > 2$

c) $f(y) = y \exp(-\frac{1}{2}y^2)$, $y > 0$