**1st exercises for SIM'2018**


**Ex. 1**
Show with Venn diagram for events $A$ and $B$ that
a) $A = AB^C + AB$
b) $A \cup B = AB^C \cup A^C B \cup AB$
c) Use axiom 1.3 from material to items a) (for $A$ and $B$) and b), and derive the addition rule for two
events in Eq. 1.4: $P(A \cup B) = P(A) + P(B) - P(AB)$.

**Ex. 2**
Let's assume that $P(A|B) = P(A|B^C)$. Show that then $A \perp\!\!\!\perp B$.

**Ex. 3**
Show that $P(A) \leq 1 - P(A^C \cap B^C) \leq P(A) + P(B)$. Hints: Total probability says that $P(S) = 1$, $A \cup A^C = S$. You can use Venn diagrams for certain steps.

**Ex. 4**
There is 5 white and 10 black balls in a bowl. Ball is lifted, color checked, and returned to bowl. This
is done 10 times. What is the probability to receive a) at least one white ball, b) five white balls?

**Ex. 5**
Prove that $E(aU + b) = aE(U) + b$ for random variable $U$ and constants $a, b$. Use Eq. 1.17.

**Ex. 6**
Compute $E(Y)$, when distribution for $Y$ is
a) $f(y) = \frac{1}{2} \exp(-|y|)$, $y \in \mathbb{R}$
b) $f(y) = 8/y^3$, $y > 2$
c) $f(y) = y \exp(-\frac{1}{2}y^2)$, $y > 0$