2nd exercises for SIM'2016

Ex. 1
a) Derive log-likelihood function for model where $Y_1, \ldots, Y_n$ are i.i.d and follow Poisson distribution $\mathcal{P}(\lambda)$.
b) Make figure of $\mathrm{l}(\lambda)$ in cases where (i) $n = 10$ and $\overline{y} = e$, (ii) $n = 10$ and $\overline{y} = 25$.

Ex. 2
a) Formulate maximum likelihood equations for n i.i.d observations from Poisson distribution.
b) Derive maximum likelihood estimate for parameter λ in ML equations in case a)

Ex. 3
a) Show that mean $\overline{y}$ is the MLE for μ when Y_i are i.i.d and follow $\mathcal{N}(\mu, 1)$.
b) Make figure of $\mathrm{l}(\mu)$ when $\overline{y} = 3$ and (i) $n = 20$, (ii) $n = 40$.

Ex. 4
We have 10 observations from i.i.d. Poisson model and $\overline{y} = 3.4$. Construct 95 % confidence interval for λ using asymptotic result in Eq. (2.7).

Ex. 5
We have two sets of stars, A and B, and we have observed their magnitudes. In set A we have 61 observations with mean magnitude -24.4, standard deviation is 3.9. In set B there is 71 observations, mean magnitude is -23.2 with standard deviation of 3.8. Test if the expected magnitudes could be the same in the groups.

Ex. 6
Test if hair color and color of eyes are independent. The data from 95 persons is

hair \eyes	blue	brown	other
blonde	32	14	6
dark	12	22	9

Note to Exs. 5 and 6. If your software cannot compute the cdf and inverse cdf for t-distribution you can use $\mathcal{N}(0, 1)$ instead. If you cannot compute χ^2-distribution, you can approximate it with $\mathcal{N}(\kappa, 2\kappa)$, where κ is the degrees of freedom. If you cannot compute cdf and inverse cdf values for $\mathcal{N}(0, 1)$, change software.