

## 1st exercises for SIM'2016

### Ex. 1

Show with Venn diagram for events  $A$  and  $B$  that

a)  $A = AB^C + AB$

b)  $A \cup B = AB^C \cup A^C B \cup AB$

c) Use axiom 1.3 from material to items a) (for  $A$  and  $B$ ) and b), and derive the addition rule for two events in Eq. 1.4:  $P(A \cup B) = P(A) + P(B) - P(AB)$ .

### Ex. 2

Let's assume that  $P(A|B) = P(A|B^C)$ . Show that then  $A \perp\!\!\!\perp B$ .

### Ex. 3

Show that  $P(A) \leq 1 - P(A^C \cap B^C) \leq P(A) + P(B)$ . Hints: Total probability says that  $P(S) = 1$ ,  $A \cup A^C = S$ . You can use Venn diagrams for certain steps.

### Ex. 4

There is 5 white and 10 black balls in a bowl. Ball is lifted, color checked, and returned to bowl. This is done 10 times. What is the probability to receive a) at least one white ball, b) five white balls?

### Ex. 5

Prove that  $E(aU + b) = aE(U) + b$  for random variable  $U$  and constants  $a, b$ . Use Eq. 1.17.

### Ex. 6

Compute  $E(Y)$ , when distribution for  $Y$  is

a)  $f(y) = \frac{1}{2} \exp(-|y|)$ ,  $y \in \mathbb{R}$

b)  $f(y) = 8/y^3$ ,  $y > 2$

c)  $f(y) = y \exp(-\frac{1}{2}y^2)$ ,  $y > 0$

### Ex. 7

Let  $U$  have uniform distribution between  $(-1, 1)$ , so  $f_U(u) = 1/2$ . What is the distribution of transformed variable  $V = U^2$ ?

### Ex. 8

Download the datafile `two-variable.dat` and make report (including, e.g., statistics of the two variables (columns), dependence, plots...)