Appendix

1.1 Normal and related distributions

Pdf's, cdf's and inverse cdf's for normal, t, χ^2, and $\mathcal{F}$-distributions, formulated using special functions.

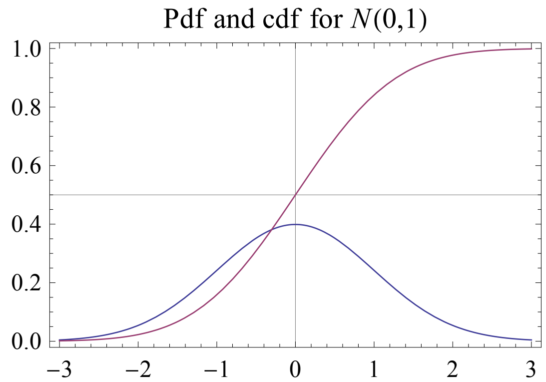
Standard normal distribution

$$\mathrm{f}(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) \tag{1.1}$$

$$\mathrm{F}(y) = \int_{-\infty}^{y} \mathrm{f}(x)dx = \frac{1}{2}\left(1 - \mathrm{erfc}\left(-\frac{y}{\sqrt{2}}\right)\right) \tag{1.2}$$

$$\mathrm{F}^{-1}(p) = \{y : \mathrm{F}(y) = p\} = -\sqrt{2}\,\mathrm{erfc}^{-1}(2p) \tag{1.3}$$

where erfc is the complementary error function, and erfc^{-1} its inverse function.



Standard normal distribution, pdf and cdf.

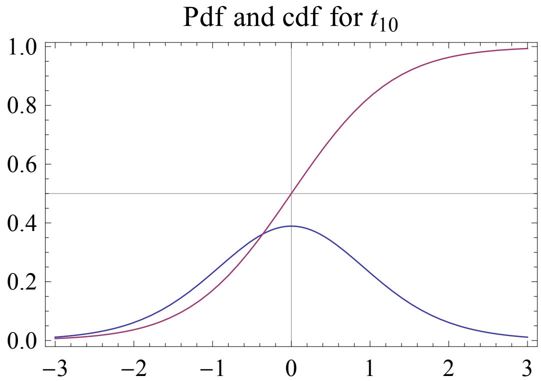
t-distribution

$$\mathrm{f}(y) = \frac{1}{\sqrt{\kappa}\,\mathrm{B}(\kappa/2, 1/2)} \left(\frac{\kappa}{\kappa + y^2}\right)^{\frac{\kappa+1}{2}} \tag{1.4}$$

$$\mathrm{F}(y) = \int_{-\infty}^{y} \mathrm{f}(x)dx = \frac{1}{2}\,\mathrm{I}\left(\frac{\kappa}{y^2+\kappa}, \frac{\kappa}{2}, \frac{1}{2}\right), \text{ if } y \leq 0, \text{ and} \tag{1.5}$$

$$\frac{1}{2}\left(1 + \mathrm{I}\left(\frac{y^2}{y^2+\kappa}, \frac{1}{2}, \frac{\kappa}{2}\right)\right), \text{ if } y > 0$$

where κ is the degrees of freedom for the distribution, B is the Euler beta function, and $\mathrm{I}(z, a, b)$ is the regularized incomplete beta function.



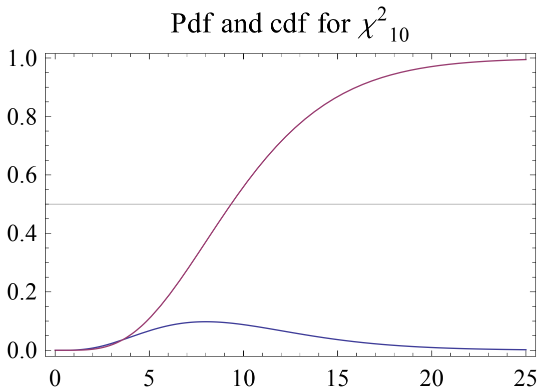
Student's t-distribution with 10 degrees of freedom, pdf and cdf.

χ^2-distribution

$$\mathrm{f}(y) = \frac{2^{-\kappa/2}\exp(-y/2)y^{\frac{\kappa}{2}-1}}{\Gamma\left(\frac{\kappa}{2}\right)} \tag{1.6}$$

$$\mathrm{F}(y) = \int_{-\infty}^{y} \mathrm{f}(x)dx = \mathrm{Q}\left(\frac{\kappa}{2}, 0, \frac{y}{2}\right) \tag{1.7}$$

where κ is the degrees of freedom for the distribution, Γ is the Euler gamma function, and $\mathrm{Q}(a, z_0, z_1)$ is the generalized regularized incomplete gamma function.



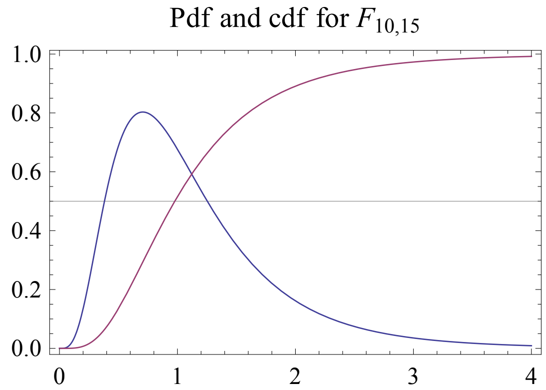
χ^2-distribution with 10 degrees of freedom, pdf and cdf.

$\mathcal{F}$-distribution

$$\mathrm{f}(y) = \frac{\kappa_1^{\kappa_1/2} \kappa_2^{\kappa_2/2} y^{\frac{\kappa_1}{2}-1} (\kappa_2 + \kappa_1 y)^{\frac{1}{2}(-\kappa_1-\kappa_2)}}{\mathrm{B}\left(\frac{\kappa_1}{2}, \frac{\kappa_2}{2}\right)} \tag{1.8}$$

$$\mathrm{F}(y) = \int_{-\infty}^{y} \mathrm{f}(x)dx = \mathrm{I}\left(\frac{y\kappa_1}{y\kappa_1 + \kappa_2}, \frac{\kappa_1}{2}, \frac{\kappa_2}{2}\right) \tag{1.9}$$

where κ_1 and κ_2 are the degrees of freedom for the distribution, B is the Euler beta function, and $\mathrm{I}(z, a, b)$ is the regularized incomplete beta function.



$\mathcal{F}$-distribution with $\kappa_1 = 10, \kappa_2 = 15$, pdf and cdf.

1.2 Matrix algebra

In what follows we introduce some simple properties of matrix algebra that should be useful with the material in this course. First, some rules regarding matrix transpose:

$$(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T \qquad (\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T \qquad (\mathbf{A}^T)^T = \mathbf{A} \tag{1.10}$$

$$(\mathbf{A}^{-1})^T = (\mathbf{A}^T)^{-1} \qquad \det(\mathbf{A}^T) = \det(\mathbf{A}) \tag{1.11}$$

$$\text{If } \mathbf{A} \text{ symmetric, then } \mathbf{A}^T = \mathbf{A} \tag{1.12}$$

$$\text{If } \mathbf{A} \text{ orthogonal, then } \mathbf{A}^T = \mathbf{A}^{-1} \text{ and } \mathbf{A}\mathbf{A}^T = \mathbf{I} \tag{1.13}$$

and matrix inverse:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \qquad (\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1} \qquad \det(\mathbf{A}^{-1}) = \det(\mathbf{A})^{-1} \tag{1.14}$$

$$\text{If } \det(A) = 0, \text{ then } \mathbf{A} \text{ is singular and cannot be inverted} \tag{1.15}$$

$$\text{If } \mathbf{A} \text{ is invertible, then columns of } \mathbf{A} \text{ are linearly independent} \tag{1.16}$$

$$\text{If } \mathbf{A} \text{ is invertible, then } \mathbf{A}^T \text{ is invertible} \tag{1.17}$$

If matrix $\mathbf{A}$ is diagonal, all the entries outside the diagonal $[\mathbf{A}]_{ii}$ are zero. Diagonal matrix can be noted by listing its diagonal elements, $\mathbf{A} = \lceil a_{11}\ a_{22}\ \cdots\ a_{nn} \rfloor$. For diagonal matrices inverse and determinant are easy to calculate:

$$\mathbf{A}^{-1} = \left\lceil \frac{1}{a_{11}}\ \frac{1}{a_{22}}\ \cdots\ \frac{1}{a_{nn}} \right\rfloor \tag{1.18}$$

$$\det(\mathbf{A}) = \prod_i a_{ii} \tag{1.19}$$

Basic rules regarding expectation and covariance operators with matrices:

$$\mathrm{E}(\mathbf{A}Y) = \mathbf{A}\,\mathrm{E}(Y) \qquad \mathrm{cov}(\mathbf{A}Y) = \mathbf{A}\,\mathrm{cov}(Y)\,\mathbf{A}^T \tag{1.20}$$