Enumerating Teams in First-order Team Logics

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Joint work with Arne Meier, Fabian Müller and Heribert Vollmer

Team Logics

- here: first-order team logics
- teams: sets of assignments
- evaluate FO-formulae over teams

- (negated) FO-atoms: satisfied by all assignments in team

$\rightsquigarrow$ flatness

- $\wedge$ as usual
- split junction: $\mathcal{A} \models_X \varphi_1 \vee \varphi_2$
  - there are $X_1 \cup X_2 = X$ such that $\mathcal{A} \models_{X_1} \varphi_1$ and $\mathcal{A} \models_{X_2} \varphi_2$
- no arbitrary negation

additional atoms for interesting team properties

$=(\overline{x}, y)$	dependence	functional dependence
$\overline{x} \perp_{\overline{z}} \overline{y}$	independence	$\overline{x}$ indep. of $\overline{y}$ wrt. $\overline{z}$
$\overline{x} \subseteq \overline{y}$	inclusion	values of $\overline{x}$ subset of values of $\overline{y}$

$\overline{x} \perp \overline{y}$: s_1, s_2 imply s_3 agreeing with s_1 on $\overline{x}$ and with s_2 $\overline{y}$

not flat, interesting team properties (databases etc.)

here: data complexity

φ fixed, given $\mathcal{A}$
Is there a team $X \neq \emptyset$ with $\mathcal{A} \models_X \varphi$?
decision problem

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decision problem

similarly: counting (data) complexity

φ fixed, given $\mathcal{A}$
How many teams $X \neq \emptyset$ satisfy $\mathcal{A} \models_X \varphi$?

Previous Research

- $\mathrm{FO}(\perp) = \mathrm{FO}(=(\dots)) = \Sigma^1_1 = \mathrm{NP}$ [Kontinen, Väänänen (2009)]
- $\mathrm{FO}(\subseteq) = \mathrm{GFP}^+ = \mathrm{LFP} = \mathrm{P}$ [Galliani, Hella (2013)]
- $\#\mathrm{FO}(\perp) = \#\mathrm{NP}$ [1]
- $\#\mathrm{FO}(\subseteq) \subseteq \mathrm{TotP} \subsetneq \#\mathrm{P}$ unless $\mathrm{P} = \mathrm{NP}$ [1]
- $\#\mathrm{FO}(=(\dots))$ contains #NP-complete problem, does not seem to capture #NP [1]

[1]: H., Kontinen, Müller, Vollmer, Yang (2019)

$\mathrm{FO}(\subseteq)$ in P

$\mathrm{FO}(\perp)$, $\mathrm{FO}(=(\ldots))$ NP-complete

Enumeration

- enumerate all solutions to a problem without duplicates
- potentially exponential number of solutions
- different notion of efficient computation needed

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polynomial delay

(+ pre- and post-computation)

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DelP

- hard decision problems $\rightsquigarrow$ DelP unlikely
- theory for hard enumeration problems needed
- enumeration analogue of polynomial hierarchy PH ...
- ... and corresponding notion of hardness

[Creignou, Kröll, Pichler, Skritek, Vollmer (2020)]

$\mathsf{DelNP} = \mathsf{DelP}^{\mathsf{NP}}$

oracle machine, query length polynomial

generalization to polynomial hierarchy

$\mathsf{DelP} = \mathsf{DelNP}$ implies $\mathsf{P} = \mathsf{NP}$

$E_1 \leq_D E_2$: $E_1 \in \mathrm{DelP}^{E_2}$

oracle access to E_2, can access *next* solution

query length polynomial

DelP and DelNP closed under $\leq_D$

Results

Problem: $\textsc{E-Sat}_\varphi^{\mathsf{team}}$
Input: structure $\mathcal{A}$
Solutions: all satisfying teams $X \neq \emptyset$ of φ in $\mathcal{A}$

- also variants for solutions that are
- inclusion maximal: $\textsc{E-MaxSat}$
- of maximum cardinality: $\textsc{E-CMaxSat}$
- analogously for minimal/minimum

$\textsc{E-Sat}^{\mathsf{team}}_{\varphi} \in \mathsf{DelNP}$

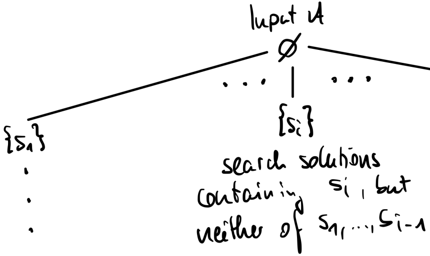
main ingredient: $\textsc{VerifyTeam}_{\varphi}, \textsc{ExtendTeam}_{\varphi} \in \mathsf{NP}$

Problem:	$\textsc{VerifyTeam}_{\varphi}$
Input:	structure $\mathcal{A}$, team X
Question:	Does X satisfy φ in $\mathcal{A}$?

Problem:	$\textsc{ExtendTeam}_{\varphi}$
Input:	structure $\mathcal{A}$, teams X, Y
Question:	Is there $X' \supsetneq X$ that satisfies φ in $\mathcal{A}$ such that $X' \cap Y = \emptyset$?

$\text{E-S}{\small AT}_{\varphi}^{\mathsf{team}} \in \mathsf{DelNP}$

main ingredient: $\text{V}{\small ERIFY}\text{T}{\small EAM}_{\varphi}, \text{E}{\small XTEND}\text{T}{\small EAM}_{\varphi} \in \mathsf{NP}$



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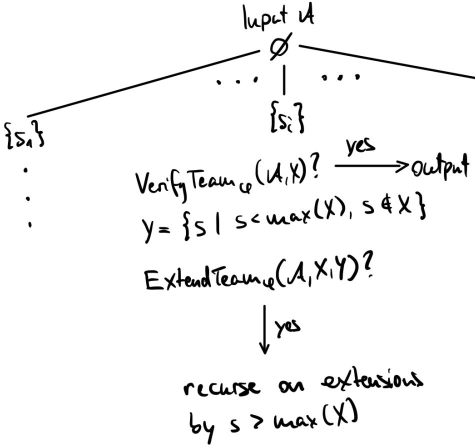

E-CMinSat and E-CMaxSat?

- compute minimum/maximum cardinality in P^{NP}
- modify $\textsc{VerifyTeam}_\varphi$ and $\textsc{ExtendTeam}_\varphi$
- check cardinality of solutions

DelNP

E-MinSat?

- again modify E-Sat-algorithm
- stop recursion after output
  extensions of solutions are not minimal
  branches with a solution also have a minimal one

DelNP

	$\subseteq$	$=(\dots),\perp$
E-SAT	$\in$ DelNP	$\in$ DelNP
E-MAXSAT		
E-CMAXSAT	$\in$ DelNP	$\in$ DelNP
E-MINSAT	$\in$ DelNP	$\in$ DelNP
E-CMINSAT	$\in$ DelNP	$\in$ DelNP

	$\subseteq$	$=(\ldots),\bot$
E-Sat	$\in$ DelNP	$\in$ DelNP
E-MaxSat		
E-CMaxSat	$\in$ DelNP	$\in$ DelNP
E-MinSat	$\in$ DelNP	$\in$ DelNP
E-CMinSat	$\in$ DelNP	$\in$ DelNP

Better algorithm for FO($\subseteq$)?

main ingredients

- unique maximal satisfying team (by union closure)
- MaxSubTeam computable in polynomial time

Problem:	$\textsc{MaxSubTeam}_\varphi$
Input:	structure $\mathcal{A}$, team X
Output:	maximal subteam of X satisfying φ in $\mathcal{A}$

Better algorithm for FO($\subseteq$)?

main ingredients

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- MaxSubTeam computable in polynomial time

$$\textsc{E-MaxSat}, \textsc{E-CMaxSat} \in \mathsf{FP}$$

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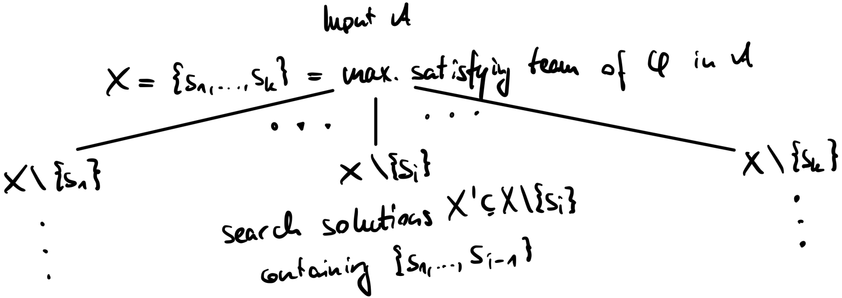
Next: E-SAT

Better algorithm for FO(⊆)?



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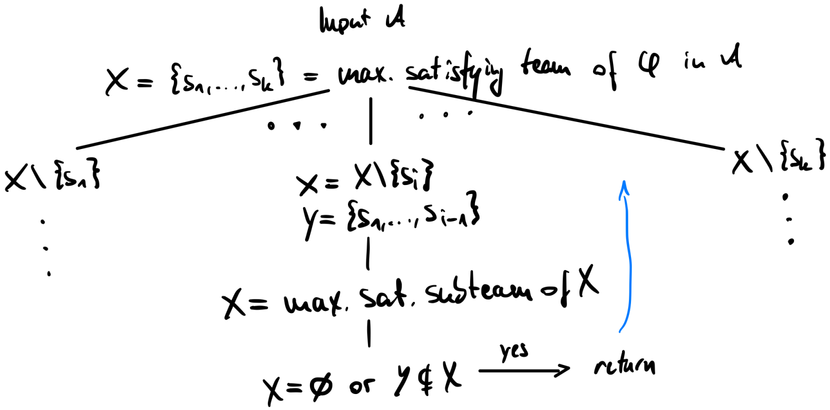

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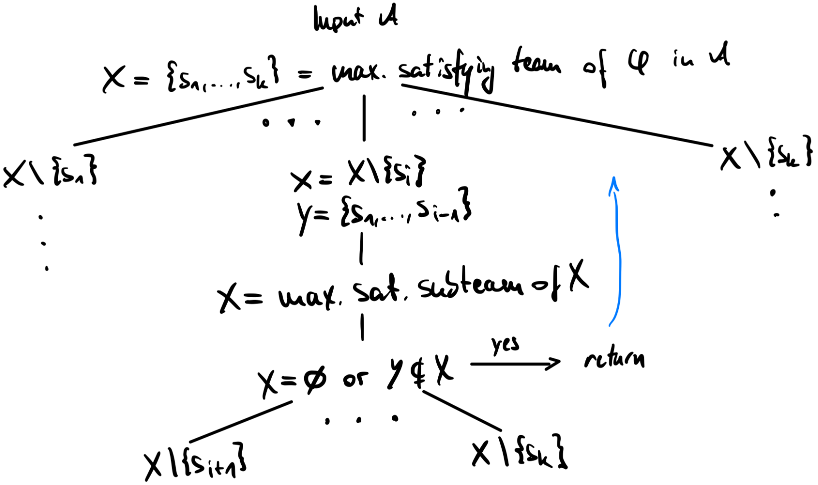

Better algorithm for FO($\subseteq$)?

$$\textsc{E-Sat} \in \mathsf{DelP}$$

	$\subseteq$	$=(\dots),\bot$
E-SAT	$\in$ DelP	$\in$ DelNP
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E-CMAXSAT	$\in$ FP	$\in$ DelNP
E-MINSAT	$\in$ DelNP	$\in$ DelNP
E-CMINSAT	$\in$ DelNP	$\in$ DelNP

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	$\subseteq$	$=(\ldots),\perp$
E-SAT	$\in$ DelP	DelNP-complete
E-MAXSAT	$\in$ FP	DelNP-hard
E-CMAXSAT	$\in$ FP	DelNP-complete
E-MINSAT	$\in$ DelNP	DelNP-complete
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Remaining cases for FO($\subseteq$)

main ingredients

- connection to myopic formulae
- $\forall \overline{x}\ (R(\overline{x}) \to \psi(R, \overline{x}))$, R only occurs positively
- can express satisfiability of Horn-formulae

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E-CMinSat DelNP-complete

- decision problem already hard
- reduction from IS via MaxZerosDualHorn

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E-MinSat?

- corresponding variant of IS not hard

Remaining cases for FO($\subseteq$)

main ingredients

- connection to myopic formulae
- $\forall \overline{x}\ (R(\overline{x}) \to \psi(R, \overline{x}))$, R only occurs positively
- can express satisfiability of Horn-formulae

E-MinSat? open!

- corresponding variant of IS not hard
- possibly: use hardness of E-MaxDualHorn

[Kavvadias, Sideri, Stavropoulos (2000)]

	$\subseteq$	$=(\dots),\bot$
E-SAT	$\in$ DelP	DelNP-complete
E-MAXSAT	$\in$ FP	DelNP-hard, $\in$ DelNP^{+}
E-CMAXSAT	$\in$ FP	DelNP-complete
E-MINSAT	DelNP-complete?	DelNP-complete
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Thank you!