Global bifurcation analysis of ecological models

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Abstract

In this seminar I will talk about the development and application of techniques for the detection and continuation of global bifurcations in ecological food web models. The work is in collaboration with Bob Kooi (VUA Netherlands), Yuri Kuznetsov (UU Netherlands), Martin Boer and Lia Hemerik (WUR Netherlands) and Eusebius Doedel (Concordia Canada).

Food web models of ordinary differential equations can display different types of qualitative behaviour for different sets of parameters (equilibria, periodic cycles, extinctions, chaos, multistability). The transitions between these different types of behaviour in parameter space are known as bifurcations. The detection and continuation of so-called local bifurcations requires only information about the local state and parameter space of the involved equilibrium or limit cycle, and can be done successfully by using standard software packages, like AUTO, Content, Locbif, etc.

Some biologically interesting phenomena are linked not to local bifurcations, but to global bifurcations, where one or more equilibria and/or limit cycles are connected to themselves or to each other in state space. To detect and successfully continue these global bifurcations in food web models we developed numerical methods as part of the AUTO package.

First I will discuss a simple 2D example of a global bifurcation: the occurrence of "overharvesting" in ODE models with Allee effects. In these models the total biomass of a population needs to be above a certain threshold in order for it to grow, i.e. there exist two attractors (*bistability*). These models show regions in parameter space where there is coexistence between predator and prey, and also regions where there are limit cycles, much like the standard models that display the "paradox of

enrichment". However, in a large part of parameter space there is extinction of predator and prey. The extinction boundary in parameter space is a point-to-point connecting orbit, which marks the point where the positive attractor of the system is destroyed and the bistability disappears.

Then, I will discuss new proposed methods for the numerical continuation of point-to-cycle and cycle-to-cycle connecting orbits in three-dimensional autonomous ODEs using projection boundary conditions. In this approach, the projection boundary conditions near the cycle are formulated using an eigenfunction of the associated adjoint variational equation, and hence costly and numerically unstable computations of the monodromy matrix are avoided. The equations for the eigenfunction are included in the defining boundary-value problem, and as a result the whole system can be implemented in AUTO using the standard features of the software.

Finally I will look at the application of these techniques to food web models, for instance the Rosenzweig-MacArthur model. It is found that global bifurcations are linked to several phenomena occurring in these models, a.o. the disappearance of chaos. It is proposed that global bifurcation analysis is vital for a proper understanding of ODE food web models, and some future developments are discussed.

Keywords: Bistability, food web modelling, global bifurcation, numerical methods, separatrix

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George van Voorn is from The Netherlands and studied biology at the University of Utrecht, with interimships at the Bioinformatics Group of Pauline Hogeweg and in Neurobiology. Currently he's doing a PhD at the Dept. of Theoretical Biology of Bas Kooijman, Vrije Universiteit of Amsterdam, until April 2009.