# The growth of the infinite long-range percolation cluster and an application to spatial epidemics

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I will consider long-range percolation on $\mathbb{Z}^d$, where the probability that two vertices at distance $r$ are connected by an edge is given by $p(r) = 1 - \exp[-\lambda(r)] \in (0,1)$ and the presence or absence of different edges are independent. Here $\lambda(r)$ is a strictly positive, non-increasing regularly varying function. I will show how the asymptotic growth of the number of vertices that are within graph-distance $k$ of the origin, $|\mathcal{B}_k|$, for $k \to \infty$ depends on $\lambda(r)$. Conditioned on the origin being in the (unique) infinite cluster, non-empty classes of non-increasing regularly varying $\lambda(r)$ are identified, for which respectively

- $|\mathcal{B}_k|^{1/k} \to \infty$ almost surely,
- there exist $1 < a_1 < a_2 < \infty$ such that $\lim_{k\to\infty} \mathbb{P}(a_1 < |\mathcal{B}_k|^{1/k} < a_2) = 1$,
- $|\mathcal{B}_k|^{1/k} \to 1$ almost surely.

This result can be applied to spatial SIR epidemics. In particular, regimes are identified for which the basic reproduction number, $R_0$ (which is an important quantity for epidemics in unstructured populations) may have a useful counterpart in spatial epidemics.

This talk is based on:


- P. Trapman (2010), The growth of the infinite long-range percolation cluster, Annals of Probability 38, 1583-1608 .

- S. Davis, P. Trapman, H. Leirs, M. Begon and J.A.P. Heesterbeek (2008), The abundance threshold for plague as a critical percolation phenomenon, Nature 454, 634-637.