LIMIT THEOREMS FOR SEQUENCES OF RECORDS

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Abstract

Let $\{X_k, k \geq 1\}$ be a sequence of independent random variables, $\alpha = \{\alpha_k, k \geq 1\}$ be positive real numbers, and F be a continuous distribution function. Assume that the distributions of random variables X_k are such that $P(X_k < x) = (F(x))^{\alpha_k}$. Such a sequence of random variables is called F^α-scheme. Define the number of records $\mu(n)$ in the sequence $\{X_k\}$ up to moment n as follows

$$\mu(n) = \sum_{k=1}^{n} \mathbb{I}_k,$$

where $\mathbb{I}_1 = 1$, $\mathbb{I}_k = \mathbb{I}(X_k > \max(X_1, X_2, \ldots, X_{k-1}))$, $k \geq 2$

An important fact of the theory of records is the statistical independence of these indicators. It is also known that $\mathbb{P}(I_n = 1) = \dfrac{\alpha_n}{A_n}$, where $A_n = \sum\limits_{k=1}^{n} \alpha_k$. In some cases, it is possible to express the almost sure asymptotic behavior of $\{\mu(n)\}$ in terms of the sequence $\{A_n\}$. For example,

$$\lim_{n\to\infty} \frac{\mu(n)}{\ln A_n} \to C \quad \text{exists almost surely if} \quad \lim \frac{\alpha_n}{A_n} \quad \text{exists},$$

where C is a nonrandom constant that depends on $\lim \frac{\alpha_n}{A_n}$ (see P. Doukhan, O. I. Klesov, and J. G. Steinebach, 2015).

Some new asymptotic results will be presented in the talk. Below is one of them.

Theorem. Let $0 < p_1 < p_2 < \cdots < p_m < 1, m \geq 1$, be all the partial limits of the sequence $\dfrac{\alpha_n}{A_n}$ and $\Delta_i := \left(\dfrac{p_{i-1} + p_i}{2}, \dfrac{p_i + p_{i+1}}{2}\right)$, $i = \overline{1, m}$, where $p_0 := 0$ and $p_{m+1} := 1$. Assume that

$$\tau_i := \lim_{n\to\infty} \frac{\left|\left\{k \in \mathbb{N} : \ k < n, \dfrac{\alpha_k}{A_k} \in \Delta_i\right\}\right|}{n} \qquad \text{exists for all} \quad i = \overline{1, m}.$$

Then:

$$\frac{\mu(n)}{\ln(A_n)} \to -\frac{\sum\limits_{i=1}^{m} \tau_i p_i}{\sum\limits_{i=1}^{m} \tau_i \ln(1 - p_i)} \ a.s.$$

Keywords: records, F^α-scheme, limit theorems.

References

P. Doukhan, O. I. Klesov, and J. G. Steinebach (2015) Strong Laws of Large Numbers in an F^α-Scheme. In: *Mathematical Statistics and Limit Theorems, Festschrift in Honour of Paul Deheuvels*, (eds.: M. Hallin, D.M. Mason, D. Pfeifer, J.G. Steinebach), Springer International Publishing, Switzerland, 287–303.