Astrofysiikan peruskurssi I – Kaavakokoelma

- Säteilyn perusmääritelmiä:

$$I_\nu(\theta,\phi) = \frac{dE_\nu}{dt\cos\theta dA d\nu d\omega}$$

$$F_\nu = \int_\Omega I_\nu(\theta,\phi)\cos\theta d\omega$$

$$J_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi) d\omega$$

$$H_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi)\cos\theta d\omega = \frac{F_\nu}{4\pi}$$

$$K_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi)\cos^2\theta d\omega = \frac{c}{4\pi}P_\mathrm{R}$$

$$u = \frac{1}{c}\int_\Omega I_\nu(\theta,\phi) d\omega$$

- Mustan kappaleen säteily:

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$\lambda_\mathrm{max} T = 0.289782 \text{ cmK}$$

$$F_\mathrm{tot} = \pi\int_0^\infty B_\nu d\nu = \sigma T_\mathrm{eff}^4, \quad \sigma = 5.669\cdot 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$$

- Säteilyn emissio ja absorptio:

$$j_\nu = \frac{1}{4\pi}\frac{dE_\nu}{dt d\nu dm}$$

$$k_\nu = \frac{\kappa_\nu}{\rho}$$

$$\tau_\nu(x) = \int_0^x \rho(x')k_\nu(x')dx'$$

Kirchoffin laki: $\frac{j_\nu}{k_\nu} = B_\nu(T)$

- Säteilynkuljetus:

$$\cos\theta\frac{dI_\nu(\tau_\nu,\theta)}{d\tau_\nu} = I_\nu(\tau_\nu,\theta) - S_\nu(\tau_\nu), \qquad S_\nu = \frac{j_\nu}{k_\nu}$$

$$\cos\theta\frac{dI_\nu}{d\tau_\nu} = I_\nu - \left(\frac{k_\nu}{k_\nu+\sigma_\nu}B_\nu + \frac{\sigma_\nu}{k_\nu+\sigma_\nu}J_\nu\right)$$

$$I_\nu(\tau_\nu,\theta) = \int_{\tau_\nu}^\infty S_\nu(\tau'_\nu)e^{-(\tau'_\nu-\tau_\nu)\sec\theta}\sec\theta d\tau'_\nu, \qquad \sec\theta = 1/\cos\theta$$

$$J_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(\tau'_\nu)E_1(|\tau'_\nu-\tau_\nu|)d\tau'_\nu$$

$$F_\nu(\tau_\nu) = 2\pi\int_{\tau_\nu}^\infty S_\nu(\tau'_\nu)E_2(\tau'_\nu-\tau_\nu)d\tau'_\nu - 2\pi\int_0^{\tau_\nu} S_\nu(\tau'_\nu)E_2(\tau_\nu-\tau'_\nu)d\tau'_\nu$$

Integraalieksponenttifunktio: $E_n(x) = \int_1^\infty \frac{e^{-xy}}{y^n}dy$

- Kaasumaisen tilan fysiikkaa:

  $PV = NRT, \quad P = NkT, \quad P = \frac{\rho kT}{m}$

  $P = \frac{1}{3}m\overline{v^2}, \quad \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$

  Adiabaattinen muutos: $PV^\gamma = \text{vakio}, \quad TV^{\gamma-1} = \text{vakio}, \quad P^{1-\gamma}T^\gamma = \text{vakio}$

- Maxwellin nopeusjakautuma, Boltzmannin virityskaava sekä Ionisaatioyhtälö:

  $dN(v_x, v_y, v_z) = N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)} dv_x dv_y dv_z$

  $dN(v) = 4\pi v^2 N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} dv$

  $\frac{N_n}{N_m} = \frac{g_n}{g_m} e^{-(E_n - E_m)/kT}$

  $\frac{N_n}{N} = \frac{g_n}{u(T)} e^{-\chi_n/kT}$

  $\frac{N_{i+1}N_e}{N_i} = \frac{(2\pi mkT)^{3/2}}{h^3} \frac{2u_{i+1}(T)}{u_i(T)} e^{-\chi_i/kT}$

  $\frac{N_{i+1}P_e}{N_i} = 0.331 T^{5/2} \cdot \frac{2u_{i+1}(T)}{u_i(T)} 10^{-\frac{5040}{T}\chi_i}$,

  $[P_e] = \text{dyn/cm}^2$, $[T] = \text{K}$, $[\chi_i] = \text{eV}$