Function theory II, spring 2010

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What is going on?

The course is over, thanks for participation! The results of the tests are found in the register. The maximum amount of extra points from the exercises was 10 (from the two tests 24+24). The maximum amount of extra points from the exercises for a final exam ('loppukoe') is 6.

Exercises

SUGGESTIONS FOR SOLUTIONS:

Lecture notes (scetches only)

Prerequisities

Function theory I

Scope

10 op.

Type

Advanced studies.

Literature

Several books are mentioned at the lectures. However, nearly basic book on function theory (of which there are plenty!) cover most of the lectures.

Lecturer

Eero Saksman

Lectures

At least the weeks 3-4, 6-9, 11, 13, 15-17 tu 14-16 C124, we 10-13 C123, in addition exercise groups 2 hours weekly.
First lecture 19.01.

Eastern holidays 1.-7.4.

Test

Two exams. Second test is on Thursday 20.5 10-13 in room C123. The solution of Dirichlet's problem in the general domain is not included in the area of the test.

Register

Did you forget to register? Mitä tehdä.

Exercise groups

Ryhmä	Päivä	Aika	Paikka	Pitäjä
1.	Fri	10-12	B322	Jarmo Jääskeläinen

Logbook

Tuesday 19.1: general things, recalling FT 1. Analyticity of uniform limits.

Wednesday 20.1: Local invertibility of analytic maps at points of conformality. Local mapping properties.
Analytic maps are open maps. Removable singularities.

Tuesday 26.1: Removable singularities (cont.). Poles. Essential singularities. Weierstrass theorem on behaviour close to an essential singularity.

Wednesday 27.1: Analytic continuations. Laurent series. Residue at a singularity.

Tuesday 9.2: The residue theorem. Applications to integrals of rational functions.

Wednesday 10.2: The residue thm (cont.). Computation of trigonometric integrals and other examples. The argument principle.

Tuesday 16.2: Rouche's theorem. New proof of openess of analytic functions and the fundamental theorem of algebra. Injectivity of limits of sequences of injective maps.

Wednesday 17.2: The gamma function: functional equation and the meromorphic extension to complex plane.
Gauss product formula and other basic formulas. Nonvanishing of the Gamma function. The Riemann zeta function.
Euler product formula for the zeta function.

Tuesday 23.2: Non-vanishing of zeta function for Re(s)>1. Hankel integral formula and analytic (meromorphic) continuation of the Riemann zeta function tothe whole complex plane.

Wednesday 24.2: The Riemann functional equation. Discussion of Riemann hypothesis and prime number theorem.
Proof of (one direction) of the exact relation between the Riemann hypothesis and prime number theorem.

Tuesday 2.3: Normal families. Montel's theorem.

Wednesday 3.3: Conformal equivalence of domains. Conformal bijections of the unit disc onto itself. Simply connected domains revisited.
The Riemann mapping theorem.

Tuesday 16.3: Comments on Riemann's mapping theorem. Homotopy of paths and its basic properties.

Wednesday 17.3: Independence of the fundamental group on
the base point. Invariance in homeomorphisms of the domain. Any homotopy can be expressed as a combination on finitely many elementary transformations.
Integral of an analytic function over general continuous curve.

Tuesday 16.3: The homotopic version of Cauchy's theorem. Free homotopy. Characterizations of simply connected domains.

Wednesday 17.3: Characterizations of simply connected domains (continued). E.g., a domain is simply connected iff its boundary is connected. The fundamental group of the punctured plane.

Tuesday 13.4: Definition of harmonic functions. Laplace operator. Connection to analytic functions. Existence of the conjugate function. Regularity. Invariance under analytic change of variables.

Wednesday 14.4: Mean value theorem. (local) Sub meanvalue property. Local and strong maximun principles. Uniqueness via boundary values. The Poisson formula.

Tuesday 20.4: Fundamental properties of the poisson kernel. Solution of the Dirichlet problem for the disc. Characterization of harmonicity via the mean value
principle. Removable singularities. Harnack's inequality.

Wednesday 21.4: Harnack's principle. Reflection principles for harmonic and analytic functions. Conformal maps between annuli.

Tuesday 27.4 Subharmonic functions. Maximum principles. Characterization via a maximum principle. Sign of the Laplacian of a subharmonic function.

Wednesday 28.4 (last lecture) Subharmonic functions (continued). Properties of Perron families.
Discussion of possible further topics in complex analysis.

Wednesday 12.05 (two extra lectures) Perron's method for solving the Dirichlet's problem. Barriers. Solvability for simply connected domains. Solvability for domains whose each boundary component consists of more than one point. Definition and existence of the Green function.