Introduction to Hopf algebras and representations, spring 2011

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Lecturer

Kalle Kytölä

Scope

7 cu.

Type

Advanced studies

Prerequisites

Algebra I and a good command of linear algebra

Lectures

The current version of the lecture notes is: Hopf_algebras_and_representations.pdf. This essentially consist of corrected and slightly reorganized versions of the sketches below. Comments and remarks about misprints are welcome!

Exam

Tuesday 17.5.2011 12:00-16:00

Bibliography

- Kassel & Rosso & Turaev : "Quantum groups and knot invariants", Panoramas et Synthèses, 1997 (a nice booklet)
- Kassel : "Quantum groups", Graduate Texts in Mathematics, 1995 (a textbook)
- Klimyk & Schmudgen : "Quantum groups and their representations", Springer, 1995 (a textbook)

Registration

Did you forget to register? What to do.

Exercise groups

Group	Day	Time	Place	Instructor
1.	Thursday	12-14	B322	Antti Kemppainen

Hopf algebras and representations

Hopf algebras were first introduced in algebraic topology, but they have since then found successful applications to a variety of different topics ranging from combinatorics to mathematical physics.

A natural motivation for the definition of Hopf algebras comes from viewing them simply as (associative unital) algebras with the additional structure that is needed for (1) making tensor product of representations still a representation and (2) making the space of linear maps between representations still a representation. In particular, groups and Lie algebras provide the first examples of Hopf algebras. More interesting Hopf algebras arise for example as deformations of semisimple Lie algebras.

Certain braided, non-commutative, non-cocommutative Hopf algebras are affectionately called “quantum groups”. They were introduced in the early 80’s by Drinfel’d and Jimbo, and they turned out to have particularly remarkable applications. Representations of quantum groups have proven relevant for such different topics as integrable systems of statistical mechanics, monodromy of solutions to differential equations from conformal field theory, invariants of knots and links and three-manifolds, among others.

The goal of this course is to provide mathematical background for a study of Hopf algebras and quantum groups, and to concretely describe representations of the simplest and most frequently encountered quantum group U_q(sl_2).

The course is suitable for students of mathematical physics or algebra. Good command of basic linear algebra and some elementary algebra is necessary (and pretty much sufficient) for following the course. The course may be particularly helpful for those interested for example in integrable statistical mechanics, in conformal field theory, in Lie algebras and representation theory, in combinatorics or in invariants of knots.

Planned contents

groups, algebras and representations;
complete reducibility of representations;
coalgebras, bialgebras and Hopf algebras;
convolution products;
duals of coalgebras and restricted duals of algebras;
braided bialgebras;
Drinfeld double;
the quantum group U_q(sl_2) and its representations;
example application (either combinatorics, knot invariants or statistical mechanics)

Exercises

- 27.1.2011: Problem sheet 1 (exercises-1.pdf), solutions (hopf_ex1_model_solutions.pdf)
- 3.2.2011: Problem sheet 2 (exercises-2.pdf), solutions (hopf_ex2_model_solutions.pdf)
- 10.2.2011: Problem sheet 3 (exercises-3.pdf), solutions (hopf_ex3_model_solutions.pdf)
- 17.2.2011: Problem sheet 4 (exercises-4.pdf), solutions (hopf_ex4_model_solutions.pdf)
- 24.2.2011: Problem sheet 5 (exercises-5.pdf), solutions (hopf_ex5_model_solutions.pdf)
- 3.3.2011: Problem sheet 6 (exercises-6.pdf), solutions (hopf_ex6_model_solutions.pdf)
- 17.3.2011: Problem sheet 7 (exercises-7.pdf), solutions (hopf_ex7_model_solutions.pdf)
- 24.3.2011: Problem sheet 8 (exercises-8.pdf), solutions (hopf_ex8_model_solutions.pdf)
- 31.3.2011: Problem sheet 9 (exercises-9.pdf), solutions (hopf_ex9_model_solutions.pdf)
- 7.4.2011: Problem sheet 10 (exercises-10.pdf), the calculation for Exercise 4c (hopf_sketch_ex_10_4c.pdf)
- 14.4.2011: Problem sheet 11 (exercises-11.pdf), solutions (hopf_ex11_model_solutions.pdf), remark for Exercise 1: see Lemma 25 in the lecture notes for unambiguous notation in this problem
- 28.4.2011: Problem sheet 12 (exercises-12.pdf), solutions (hopf_ex12_model_solutions.pdf)
- 5.5.2011: Problem sheet 13 (exercises-13.pdf), solutions (hopf_ex13_model_solutions.pdf)

Sketches of lecture notes

The current version of the lecture notes of the entire course is here: Hopf_algebras_and_representations.pdf

Older versions are still attached to the following schedule of lectures.

- 18.1.2011: Jordan canonical form (lecture_sketch-Jordan_normal_form.pdf), Representations of finite groups part 1 (contained in the file below)
- 25.1.2011: Representations of finite groups part 2 (contained in the file below), Tensor products of vector spaces (contained in the file below)
- 1.2.2011: Representations of finite groups part 3 (lecture_sketch-representations_of_finite_groups.pdf), Algebras (contained in the file below)
- 8.2.2011: Algebras & coalgebras & bialgebras & Hopf algebras (contained in the file below), Some more remarks about tensor products (lecture_sketch-tensor_products.pdf)
- 15.2.2011: Algebras etc. & convolution products & properties of the antipode (contained in the file below)
- 22.2.2011: Algebras etc. & representations & restricted dual (contained in the file below)
- 1.3.2011: Algebras etc. & restricted duals (lecture_sketch-algebras_coalgebras_etc.pdf), On quantum groups: restricted dual of the building block (contained in the file below)
- 15.3.2011: On quantum groups: Yang-Baxter equation and braided bialgebras (contained in the file below)
- 22.3.2011: On quantum groups: the Drinfeld double construction (contained in the file below)
- 29.3.2011: On quantum groups: U_q(sl_2) and the Drinfeld double of the building block (contained in the file below)
- 5.4.2011: On quantum groups: representations of U_q(sl_2), semisimplicity (contained in the file below)
- 12.4.2011: On quantum groups: semisimplicity of U_q(sl_2) for q not a root of unity, infinite dimensional Drinfeld doubles and Yang-Baxter equation (lecture_sketch-on_quantum_groups.pdf)
- 19.4.2011: Discussion of knots, links and tangles and graphical calculus in finite dimensional modules over braided Hopf algebras
- 3.5.2011: Discussion of graphical calculus in finite dimensional modules over ribbon Hopf algebras and invariants of knots, links and tangles