Topologia II, kevät 2015

Topologia II, kevät 2015 / Topology II, Spring 2015

Luennoitsija / Lecturer

Jani Lukkarinen

Laajuus / Scope

10 op.

Tyyppi / Type

Syventävä opinto / Advanced studies

Esitietovaatimukset / Prerequisites

Topologia I (metric topology)

Luentoajat / Schedule

Viikot 3-9 ja 11-18 ma 12-14 ja ti 16-18 salissa C124. Lisäksi laskuharjoituksia 2 viikkotuntia. Pääsiäisloma 2.-8.4.

Weeks 3-9 and 11-18 on Mon 12-14 and Tue 16-18 in Lecture room C124, and 2 hours of weekly tutorials. Easter break 2.-8.4.

Kurssin ilmoitustaulu / Course announcements

- The **results for the second midterm** exam are now available on the department's **results webpage**. If you wish, you can check the grading of your answers from the lecturer.
- The **results for the first midterm** exam are now available on the department's **results webpage.** If you wish, you can check the grading of your answers from the lecturer.
- Problem sheets can now be downloaded from below.
- Lecture diary will also be updated below.

Ilmoittaudu kurssille / Registration link

Unohditko ilmoittautua? Katso ohjeet täältä!

Did you forget to register? What to do?

Kurssikuvaus / Contents

Topologia II on matematiikan syventävien opintojen valinnainen kurssi, joka sopii mainiosti kaikille matematiikan ja soveltavan matematiikan opiskelijoille. Erityisen suositeltava se on mm. algebran ja topologian sekä matemaattisen logiikan linjoilla.

Kurssilla opiskellaan yleistä topologisten avaruuksien teoriaa, jossa lähtökohtana ovat avaruuden avoimet joukot (eli topologia) sellaisenaan – ilman että ne määriteltäisiin esimerkiksi metriikan avulla kuten kurssilla Topologia I.

Sisältöä:

- topologiset avaruudet
- topologioiden kannat
- topologioiden indusointi kuvausten avulla
- relatiivitopologia, tulotopologia ja tekijätopologia
- avaruuksien erotteluominaisuudet, mm. Hausdorff-ominaisuus
- avaruuksien numeroituvuusominaisuudet
- yhtenäisyys
- kompaktius ja kompaktisointi
- metristys
- kuvausten jatkuva jatkaminen

Contents:

- *topological spaces and continuity*
- *bases for topologies*

- *inducing and coinducing topologies using a family of maps*
- *relative, product, and quotient topologies*
- *separation axioms, Hausdorff spaces*
- *countability properties of topologies*
- *connectedness*
- *compactness and compactifications, Tychonoff's theorem*
- *metrization, completeness, Baire's theorem*
- *continuous extensions: Urysohn's lemma and Tietze's extension theorem*

Suorittaminen / Exams and grading

There are two midterm exams:

- The first course exam will be on **Thursday, March 5th, at 13:00-15:00** in one of the lecture halls in Exactum. The exam covers chapters 1-8 of the Topologia II textbook: the last topic to be included is the coinduced topology. No material marked with a star "*" will be assumed to be known (this includes also the bonus exercises and some material which was mentioned during the lectures, such as nets and filter bases).
- The second course exam will be on **Thursday, May 7th, at 13:00-15:00** in one of the lecture halls in Exactum. The exam covers chapters 9-13, 15-16, 18-19, and 20.1-20.3. of the Topologia II textbook: the last topic to be included is the Tietze extension theorem. *There will be at least one problem related to quotient spaces (chapter 9).* No material marked with a star "*" will be assumed to be known (this includes also the bonus exercises and some material which was mentioned during the lectures, such as the Kuratowski's embedding theorem and completetions of metric spaces).

The maximum points from the two midterm exams are 24+24 points. A necessary condition for passing the course is to get at least 8 points from each of the exams. The extra points (max. 8) from tutorials are added to the total at the end of the course.

Kirjallisuus / Bibliography

Kurssilla seurataan oppikirjaa / Course textbook

- Jussi Väisälä: *Topologia II* , 2. painos (2005), Limes ry (korjaukset / Errata)
  (Myös kirjan 1. painos (1999) käy (korjaukset). Huomaa kuitenkin, että lauseiden, harjoitustehtävien ym. kohtien numeroinnissa on paikoin pieniä eroja 2. painokseen verrattuna.)
- Translations of the terminology used in the above book: Finnish-English & English-Finnish .
- Closest English textbook equivalent: Stephen Willard, *General Topology,* Addison-Wesley (1970) and reprinted by Dover (2004)

Further reading:

- Bert Mendelson, Introduction to Topology, 3rd edition (for basics of general topology)
- John L. Kelley, General Topology
- N. Bourbaki, General Topology, part I
- Theodore W. Gamelin and Robert Everist Greene, Introduction to Topology: Second Edition (Dover Books on Mathematics, paperback, 1999).

Laskuharjoitukset / Tutorials

Ryhmä	Päivä	Aika	Paikka	Pitäjä
1.	ti	12-14	B321	Okko Kanerva

Group	Day	Time	Place	Instructor
1.	Tue	12-14	B321	Okko Kanerva

Solutions will be sent to the partipants by e-mail.

	Session on	PDF
7.	10.3.	ht-07.pdf
8.	17.3.	ht-08b.pdf
9.	24.3.	ht-09.pdf
10.	31.3.	ht-10.pdf
11.	14.4.	ht-11.pdf
12.	21.4.	ht-12.pdf
13.	29.4.	ht-13.pdf
	Session on	PDF

0.*	13.1.	ht-00.pdf
1.	20.1.	ht-01.pdf
2.	27.1.	ht-02.pdf
3.	3.2.	ht-03.pdf
4.	10.2.	ht-04.pdf
5.	17.2.	ht-05.pdf
6.	24.2.	ht-06.pdf

Luentopäiväkirja / Lecture Diary

Luennot / Lecture	Sisältö / Content
12. & 13.1.	Introduction to general topology and comparison to metric topology. Notations and set theory. Chapter 0 and sections 1-15 from chapter 1.
19. & 20.1.	History of topology (1.16). Topological base, neighborhood base and subbase. Topology generated by a collection of subsets. (Chapter 2) Definition of continuity (3.1 and 3.2).
26. & 27.1.	Continuous functions, homeomorphisms, convergence of sequences. (Chapter 3) Topology induced by one function (1-4 from chapter 4).
2. & 3.2.	Topology induced by one function (5-8 from chapter 4). Relative topology, continuity of restrictrions of functions. Embeddings and immersions. (Chapter 5) Topology induced by a family of functions (1-5 from chapter 6).
9. & 10.2.	Topology induced by a family of functions (6-9 from chapter 6). Product topology (1-8 from chapter 7).
16. & 17.2.	Product topology. Continuity of functions into product spaces. Products of functions. (9-16 & 19-20 from chapter 7) Topology coinduced by a family of functions. Identification maps. (1-7 from chapter 8).
23. & 24.2.	Identification maps (8-10 from chapter 8). Quotient topology (1-4 from chapter 9). Review of the exam material. Cantor set as a product space. (7.18*)
9. & 10.3.	Solutions to the exam questions. Quotient topology: projective spaces, collapsing a set into a point, homeomorphisms from the canonical decomposition of identification maps. (5-12 from chapter 9).
16. & 17.3.	Metrizability. Countable products of metric spaces. Completeness and Baire category theorem. Uniform convergence. Kuratowski's embedding theorem and completetions of metric spaces. (Chapter 10)
23. & 24.3.	Separation axioms, including Hausdorff, regular and normal spaces. (Chapter 11) Countability axioms N_1 and N_2 (1-11 from chapter 12).
30. & 31.3.	Lindelöf spaces and separability (12-23 from chapter 12). Connected space (1-5 from chapter 13).
13. & 14.4.	Basic properties of connected spaces. Connected components. Path-connectedness and path-components. Locally connected spaces. (6-31 from chapter 13) Compact space (1-5 from chapter 15).
20. & 21.4.	Basic properties of compact spaces, including their relation to the finite intersection property (f.i.p.) and the Boltzano-Weierstrass theorem (6-22 from chapter 15). Alexander subbase theorem and Tychonoff's theorem (Chapter 18.4). Urysohn's lemma (1-2 from chapter 19) and the Tietze extension theorem (1-3 from chapter 20). Axiom of choice and Zorn's lemma (1-5 from appendix Z *but not* the proof Z.6). **NB:** There is a misprint in the textbook at the definition of f.i.p. in section 15.8: the condition "the collection A is non-empty" should be removed. (One needs to include the empty collection to get the next theorem correctly when X is empty.) **Huom:** Kirjassa on painovirhe kohdassa 15.8: ÄLO:n määritelmästä pitää poistaa ehto "A on epätyhjä", ts. riittää, että kokoelma A on osajoukko kokoelmasta F.
27. & 28.4.	Proof of Urysohn's lemma and Urysohn's embedding theorem. Metrizability of regular N_2 spaces. (Chapter 19) Review of the material for the second exam.