Sobolev spaces, spring 2016

Teacher: Kari Astala
Scope: 10 cr
Type: Advanced studies


Topics: Weak derivatives and Sobolev Spaces; properties of Sobolev functions, such as traces, extensions etc.; applications to elliptic PDE's and calculus of variations.

Prerequisites: Basic knowledge on measure theory and Lebesgue integration (e.g. the course "Mitta ja integraali") is required. Knowledge of basics of $L^p$-spaces is necessary (e.g. as discussed in the course Reaalianalyysi I; see below).
In addition basic facts from functional analysis will be necessary, in particular elementary Hilbert space theory is needed.

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News

Dead line for project works: Wednesday May 25;
please email your completed project to both addresses: kari.astala@helsinki.fi and petri.ola@helsinki.fi

Seminar presentations: Monday May 30, starting at 9. Here is a preliminary order for presentations.

Project guidance next week:
Tuesday May 24, at 12-14 room C123

Teaching schedule

Lectures on Tuesdays 10-12 and Wednesdays 12-14 in room C123. First lecture Tuesday 19.1.2016

Easter Holiday 24.-30.3.

In the first period, weeks 3-9, there will be 4 hours lectures every week, and exercises every week (for details on these see below).

In the second period, weeks 11-18, there will weekly 2 hours lectures and 2 hours project guidance. Also some exercises will be given, but not every week.

Exams

Credit for the first part of the course, weeks 3-9, is gained by a course exam during week 10, covering the material of the first period.

The course exam will be on Thursday, March 10, at 12 - 15 (more precisely: 12.15 - 14.45). The area covered in the exam is Evans/Chapter 5 (topics covered according to lecture diary below + the additional notes on this Chapter).

Average in the course exam was 15.6 / 24. For results see the web-pages Koetulokset / Exam results.
Credit for the second part of the course, weeks 11-18, is gained by a project work and seminar presentation.

Both parts of the course are 5 cr. each.

Topics and details of the project works will be agreed on the first week of March. During the second part of the course, there will be weekly 2 hours lectures and 2 hours project guidance.

Here are (some) suggestions for project topics; details will be discussed and agreed next week (i.e. the first week of March), but if you wish to reserve some of the topics suggested, please send email to: kari.astala@helsinki.fi.

If necessary, further topic suggestions can be given next week. Also, your own suggestions for a project topic are still equally well come!

Seminar presentations: Monday May 30.

Course material

The course will mainly follow the book “Evans: Partial Differential Equation” (AMS Graduate Studies 19), Chapters 5, 6 and 8. Having access to the book will be necessary! In particular, Evans’ book will serve as the course notes, only notes of some additional material will given on these pages.

Lecture diary: In the first week we covered sections 5.1 and 5.2.1 - 5.2.2. from Evans’ book.
Second week covered sections 5.2.3 with 5.3 (approximation) and 5.4 (extensions).
Third week: Section 5.5 (Traces) and began Section 5.6 (Sobolev inequalities); covered Theorem 1.
Fourth week: Completed Section 5.6. (subsection 5.6.3 was left for a self study) and began Section 5.7. (compactness; discussed Ascoli-Arzela theorem).
Fifth week: Completed Section 5.7 and covered Section 5.8.1 (Poincare inequalities).
Sixth week: Covered Section 5.8.2 (Difference quotients) and 5.8.4 (Fourier transform methods), 5.8.3. was left for a self study. Skip Section 5.9.
Seventh week/end of period I: Covered 6.1. (elliptic equations and weak solutions) and 6.2.1 (Lax-Milgram theorem).
Eight week/beginning of second period (15.3): Completed solving elliptic PDE’s by Lax-Milgram; Introduction to Calculus of variations (Intro to Dirichlet’s principle, see notes below).
Ninth week / 22.3: Evans, Sections 8.1.2 - 8.1.3 + Weierstrass example where minimisers do not exits (c.f. notes below).
Week ten/April 5: Beginning of Section 8.2, pages 443-445; discussion on convexity (Evans, Appendix B.1)
Week eleven/April 12: Existence of minimisers; Evans/ Theorems 1 and 2 in Section 8.2 (Evans pp.445-449)
Week 13/April 20: Discussed results from Functional Analysis needed in proving existence of minimisers, gave complete proof of Dirichlet principle (Notes, section 14), discussed Section 8.2.3 briefly.
Week 14/April 26: Discussion on systems and Section 8.1.4.; last result covered was Lemma in Section 8.1.4.b/Evans, p. 440.

Here are some additional details/material that was discussed in the lectures, up to Wednesday April 20.

More information on real analysis, in particular on basics of $L^p$-spaces, can be found from lecture notes by Ilkka Holopainen attached here: Reaalialalyysi.1(2011).pdf The information on $L^p$-spaces needed can also be found e.g. in Evan’s book and its Appendices.

Also other relevant material will be used when needed. (More details on this when appropriate)

Here are the notes "Funktionaalialalyysin peruskurssi", where you find additional info on functional analysis.

Registration

Did you forget to register? What to do?
Exercises

Aleksis Koski and Petri Ola will be the course assistants. Assignments are given on Wednesdays.

Assignments

- Assignment 1
- Assignment 2
- Assignment 3
- Assignment 4
- Assignment 5
- Assignment 6
- Assignment 7
- Assignment 8
- Assignment 9

Suggestions for solutions

- Solutions 1
- Solutions 2
- Solutions 3
- Solutions 4
- Solutions 5
- Solutions 6
- Solutions 7
- Solutions 8
- Solutions 9

Exercise classes

Note: You get extra points from solved assignments as follows:

25\% = 1p, 35\% = 2p, 45\% = 3p, 55\% = 4p, 65\% = 5p and 75\% = 6p.

Your solutions to the exercises are to be returned weekly on Thursdays (of the following week) to Petri Ola.

For this there will be an envelope on the door of Petri Olas office (D329).

There will be exercise tutorials/ohjauksia available at the following times

(\textbf{Note} the new arrangements !)

<table>
<thead>
<tr>
<th>Group</th>
<th>Day</th>
<th>Time</th>
<th>Room</th>
<th>Instructor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Monday</td>
<td>10-12</td>
<td>A415</td>
<td>Aleksis Koski</td>
</tr>
<tr>
<td>2.</td>
<td>Thursday</td>
<td>10-11</td>
<td>4th floor corridor</td>
<td>Petri Ola</td>
</tr>
<tr>
<td>3.</td>
<td>Tuesday</td>
<td>12-13</td>
<td>4th floor corridor</td>
<td>Petri Ola</td>
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Course feedback

Course feedback can be given at any point during the course. Click \textit{here}. 