# Minimal surfaces, fall 2014

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### Lecturer

Ilkka Holopainen

### Scope

10 sp.

### Type

Advanced studies

### Prerequisites

Basic and intermediate studies.

Knowledge on differential and Riemannian geometry, complex analysis, and geometric measure theory would be useful but not necessary.

### Lectures

Weeks 36-42 and 44-50, Wednesday 10-12 and Thursday 10-12 in room C124.

**Last lecture on December 10!**

### Course description

From the book *A course in Minimal Surfaces* by T.H. Colding and W.P. Minicozzi II:

"Minimal surfaces date back to Euler and Lagrange and the beginning of the calculus of variations. Many of the techniques developed have played key roles in geometry and partial differential equations. Examples include monotonicity and tangent cone analysis originating in the regularity theory for minimal surfaces, estimates for nonlinear equations based on the maximum principle arising in Bernstein's classical work, and even Lebesgue's definition of the integral that he developed in his thesis on the Plateau problem for minimal surfaces."

From the Introduction of *Minimal Surfaces and Functions of Bounded Variation* by E. Giusti:

"The problem of finding minimal surfaces, i.e. of finding the surface of least area among those bounded by a given curve, was one of the first considered after the foundation of the calculus of variations, and is one which received a satisfactory solution only in recent years. Called the problem of Plateau, after the blind physicist who did beautiful experiments with soap films and bubbles, it has resisted the efforts of many mathematicians for more than a century. It was only in the thirties that a solution was given to the problem of Plateau in 3-dimensional Euclidean space, with the papers of Douglas and Rado."

This course gives an introduction to the theory of minimal surfaces. Some topics that might be discussed:

Geometry of submanifolds of $R^n$, Riemannian metrics, mean curvature of submanifolds
First variation of area/volume
Parametric and non-parametric minimal surfaces, examples
Bernstein's theorem
Plateau's problem
Functions of bounded variation and existence of minimal hypersurfaces in higher dimensions

### Exams

The course can be passed by an exam or by solving home work problems and writing an essay.

### Bibliography

T.H. Colding and W.P. Minicozzi II: *A Course in Minimal Surfaces*. Graduate Studies in Mathematics, Vol. 121, AMS, 2011.

M. Giaquinta and L. Martinazzi: *An introduction to the regularity theory for elliptic systems, harmonic maps and minimal graphs*. Edizioni della Normale, Pisa, 2012.

E. Giusti: *Minimal Surfaces and Functions of Bounded Variation*. Monographs in Mathematics, Vol 80, Birkhäuser, 1984.

W.H. Meeks and J. Pérez: The classical theory of minimal surfaces. Bull. Amer. Math. Soc. 48 (2011), 325-407

W.H. Meeks and J. Pérez: *A survey on classical minimal surface theory.*

J.C.C. Nitsche: *Lectures on Minimal Surfaces: Volume 1*, Cambridge University Press, 2011.

R. Osserman: *A survey of Minimal Surfaces*. Dover Publications, 2002.

http://www.indiana.edu/~minimal/

**Lecture notes:** I. Holopainen: Minimal Surfaces

**Lecture notes (slides):** Minimal submanifolds, Levico Terme, July 6-10, 2015.

**Lecture notes (slides):** Dirichlet problem at infinity for the minimal graph equation on Cartan-Hadamard manifolds, Sant Feliu de Guixols, June 16, 2015.

## Registration

Did you forget to register? What to do?

### Exercises

| Group | Day | Time | Place | Teacher |
|---|---|---|---|---|
| 1. | Friday | 10-12 | C124[1)] | Elefterios Soultanis |

1) Except B120 on October 3rd.

### Home work assignments

Exercise 1

Exercise 2

Exercise 3 Pg6-7.pdf

Exercise 4

Exercise 5

Exercise 6

Exercise 7

Exercise 8

Exercise 9

Exercise 10