

Statistical inversion

Statistical inverse problems

Introduction to Statistical inverse problems

The statistical inverse problems or statistical inversion is a branch of Bayesian statistics. The general starting point is the pair of random variables Y and X where Y stands for the *measurement* or the *observable* and X is the *unknown* quantity or object. The forward problem is to describe how the measurement depends on the unknown X . In Bayesian statistics we can state that the forward problem is to describe the likelihood "function" L , which is the conditional probability distribution of the measurement Y given the unknown X . Supposing that the state spaces of the measurement Y and the unknown X are regular enough, the likelihood is a regular conditional probability and therefore, can be considered as a measure valued random variable. In Bayesian statistics the inverse problem is as easy to define as the forward problem. The Bayesian statistical inverse problem is describe the *a posteriori* distribution "function" D , which is the conditional probability distribution of the unknown X given the measurement Y . Again, this is usually a measure valued random variable.

In practise, the forward problem is deduced from a *measurement or forward model* which is typically of form $Y=F(X,N)$ where F is a *forward map* and N is a external and/or internal *noise* that is assumed to be independent from the unknown X . The most common case is the (non)linear model with additive noise where $F(X,N)=A(X)+N$ for some (non)linear operator A . For this kind of measurement model the describing the likelihood is straight forward given the distribution of the noise N . The inverse problem can then be solved by the Bayes' Theorem which can be stated informally as D is *proportional to* p times L where p is the *a priori* distribution of the unknown X .

In the case the unknown and the measurement are finite dimensional objects the analysis can in many cases be reduced to the conditional probability densities and in that case the Bayesian statistical inverse problems become classical Bayesian estimation problems. The physical forward models, however, are typically infinite dimensional and the more general framework is needed.

Discretization and regularization problems

Applications

Publications

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