Geometric measure theory, fall 2016

Teacher: Ilkka Holopainen

Scope: 10 cr

Type: Advanced studies

Teaching: Weeks 36-42 and 44-50, Tuesday 12-14 and Thursday 10-12 in room C124. **Note: No lectures (nor home work classes) during weeks 43 (25-26.10) and 44 (1-3.11).**

Between 9.11 - 14.12 we will have extra classes/home work sessions on Wednesdays 12-14 in room B322.

Topics:
From the book *Geometric measure theory: A beginner's guide* by F. Morgan:

"*Geometric measure theory* could be described as differential geometry, generalized through measure theory to deal with maps and surfaces that are not necessary smooth, and applied to the calculus of variations."

The quotation above describes very well the goal of the course. This course gives an introduction to the theory of varifolds and currents that are kind of generalized surfaces. They have been used in many geometric variational problems, in particular, in connections with higher dimensional minimal surfaces.

Some topics that might be discussed:

Review of measure theory

- *measures and outer measures*
- *metric outer measures*
- *regularity of measures, Radon measures*
- *Hausdorff measure and dimension*
- *Riesz representation theorem*

Lipschitz mappings and rectifiable sets

- *extension of Lipschitz functions*
- *Rademacher's theorem*
- *area and co-area formulae*
- *rectifiable sets*
- *approximate tangent space*
- *densities*
- *the structure theorem*

Varifolds

- *general varifolds*
- *rectifiable k-varifolds*
- *first variation formula*
- *monotonicity formula and its consequences*
- *regularity theorem*

Currents

- *k-vectors and k-covectors*
- *differential forms*
- *currents (definition and basic notions)*
- *spaces of currents*
- *slicing*
- *deformation theorem*
- *isoperimetric inequality*
- *rectifiability and compactness theorems*

Mass (area) minimizing currents

- *existence*
- *properties of mass minimizing currents*
- *regularity of mass minimizing currents*

Prerequisites:

Good knowledge on measure and integration theory (courses like *Measure and integral* and *Real analysis I*). The course *Real analysis II* would be helpful but is not necessary.

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News

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Teaching schedule

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Exams

The course can be passed by an exam or preferably by giving presentations in the home work classes and writing essays.

How to pass the course by giving presentations? The plan is that a student gives two presentations (1-2 hours each) on a chosen topic and writes an essay on the topic (and, of course, follows the presentations of others).

Course material

Books

L. Evans and R. Gariepy: *Measure theory ans fine properties of functions,* CRC Press, 1992.

H. Federer: *Geometric Measure Theory*, Springer, 1969.

F. Lin and X. Yang: *Geometric Measure Theory: An Introduction*, International Press, 2002.

P. Mattila: *Geometry of sets and measures in Euclidean spaces*, Cambridge University Press,1995.

F. Morgan: *Geometric Measure Theory, A Beginner's Guide*, Academic Press, 1987. An e-book available for students of UH.

L. Simon: *Lectures on Geometric Measure Theory*, Australian National University, 1983. An updated version.

Lecture notes

P. Mattila: Currents and varifolds. Hand-written lecture notes, fall 2011.

I. Holopainen: Geometric measure theory (first 88 pages + appendix). Proof of the Deformation theorem, discussions on regularity results (stationary varifolds, mass minimizing currents) will be added later.

Registration

Did you forget to register? What to do?

Exercises

- Exercises 1 Solutions 1
- Exercises 2 Solutions 2
- 28.9.2016, Presentation on the Riesz representaton theorem by Ville Marttila (see Appendix in the lecture notes)
- 5.10.2016, Presentation on weak convergence of measures and on the compactness of Radon measures by Janne Siipola (see Appendix in the lecture notes)
- 12.10.2016, Presentation on Rademacher's theorem by Akseli Haarala
- 19.10.2016, Presentation on the area formula by Krishnan Narayanan
- 8.11.2016 (Tuesday), Presentation on the co-area formula by Otte Heinävaara
- 9.11.2016, Presentation on the Whitney extension by Valter Lillberg
- 9.11.2016, Presentation on submanifolds and mean curvature by Ville Karlsson
- 16.11.2016, Presentation on approximate tangent spaces and on rectifiable sets (part I) by Ville Marttila
- 16.11.2016, Presentation on approximate tangent spaces and on rectifiable sets (part II) by Janne Siipola

- 23.11.2016, Presentation on the monotonicity formula and isoperimetric inequality by Akseli Haarala
- 23.11.2016, Presentation on the rectifiability theorem and tangent cones (part I) by Krishan Narayanan
- 30.11.2016, Presentation on the rectifiability theorem and tangent cones (part II) by Otte Heinävaara
- 30.11.2016, Presentation on the regularity theory (part I) by Valter Lillberg
- 7.12.2016, Presentation on the regularity theory (part II) by Ville Karlsson

Exercise classes

Group	Day	Time	Room	Instructor
1.	Wednesday	14-16	C129	Ilkka Holopainen

Course feedback

Course feedback can be given at any point during the course. Click here.