On strongly minimal Steiner systems: Zilber's Conjecture, Universal Algebra, and Combinatorics

John T. Baldwin
University of Illinois at Chicago
Helsinki, Jan 28, 2019

January 6, 2019

Abstract

With Gianluca Paolini (in preparation), we constructed, using a variant on the Hrushovski dimension function, for every $k \geq 3$, 2^{μ} families of strongly minimal Steiner k-systems. We study the mathematical properties of these counterexamples to Zilber's trichotomy conjecture rather than thinking of them as merely exotic examples. In particular the long study of finite Steiner systems in reflected in results that depend on the block size k. A quasigroup is a structure with a binary operation such that for each equation $xy = z$ the values of two of the variables determines a unique value for the third. The new Steiner 3-systems are bi-interpretable with strongly minimal Steiner quasigroups. For $k > 3$, we show the pure k-Steiner systems have 'essentially unary definable closure' and do not interpret a quasigroup. But we show that for q a prime power the Steiner q-systems can be interpreted into specific sorts of quasigroups, block algebras.

We extend the notion of an (a, b)-cycle graph arising in the study of finite and infinite Stein triple systems (e.g Cameron-Webb) by introducing what we call the (a, b)-path graph of a block algebra. We exhibit theories of strongly minimal block algebras where all (a, b)-paths are infinite and others in which all are finite only in the prime model. We show how to obtain combinatorial properties (e.g. 2-transitivity) by either varying the basic collection of finite partial Steiner systems or modifying the μ function which ensures strong minimality.