On strongly minimal Steiner systems:
Zilber’s Conjecture, Universal Algebra, and
Combinatorics

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Abstract

With Gianluca Paolini (in preparation), we constructed, using a variant on the
Hrushovski dimension function, for every \( k \geq 3 \), \( 2^\mu \) families of strongly minimal
Steiner \( k \)-systems. We study the mathematical properties of these counterexamples
to Zilber’s trichotomy conjecture rather than thinking of them as merely exotic exam-
pies. In particular the long study of finite Steiner systems in reflected in results that
depend on the block size \( k \). A quasigroup is a structure with a binary operation such
that for each equation \( xy = z \) the values of two of the variables determines a unique
value for the third. The new Steiner 3-systems are bi-interpretable with strongly min-
imal Steiner quasigroups. For \( k > 3 \), we show the pure \( k \)-Steiner systems have
‘essentially unary definable closure’ and do not interpret a quasigroup. But we show
that for \( q \) a prime power the Steiner \( q \)-systems can be interpreted into specific sorts of
quasigroups, block algebras.

We extend the notion of an \((a, b)\)-cycle graph arising in the study of finite and
infinite Stein triple systems (e.g Cameron-Webb) by introducing what we call the
\((a, b)\)-path graph of a block algebra. We exhibit theories of strongly minimal block
algebras where all \((a, b)\)-paths are infinite and others in which all are finite only in the
prime model. We show how to obtain combinatorial properties (e.g. 2-transitivity) by
either varying the basic collection of finite partial Steiner systems or modifying the \( \mu \)
function which ensures strong minimality.