The mathematical theory of minimal surfaces goes back to the mid-18th century when Euler and Lagrange developed the calculus of variations. In addition to the calculus of variations, minimal surfaces are related to (and have greatly influenced on) various fields in mathematics, like complex analysis, geometric measure theory, elliptic PDEs, differential geometry to mention but a few.

In the first part of the talk I will discuss some of these connections. In the second part I will review our recent studies on the existence of minimal hypersurfaces in $M \times \mathbb{R}$, where $M$ is a complete Riemannian manifold (usually a Cartan-Hadamard manifold). This part of the talk is based on joint works with Jean-Baptiste Casteras, Esko Heinonen, and Jaime Ripoll.