On atomic decompositions for Hardy spaces associated with certain Schrödinger operators
Jacek Dziubanski
University of Wroclaw, Poland

Let $L = -\Delta + V$ be a Schrödinger operator on $\mathbb{R}^d$ with a nonnegative locally integrable potential $V$ and let $K_t(x,y)$ be the integral kernels of the semigroup $\{T_t\}_{t>0}$ generated by $-L$. The Feynman-Kac formula implies that

$$0 \leq K_t(x,y) \leq (4\pi t)^{-d/2} \exp\left(-\frac{|x-y|^2}{4t}\right).$$

Let

$$M_L f(x) = \sup_{t>0} |T_t f(x)|.$$

be the maximal function associated with $\{T_t\}_{t>0}$. We say that an $L^1(\mathbb{R}^d)$-function $f$ belongs to $H^1_L$ if $M_L f \in L^1(\mathbb{R}^d)$. Then we set $\|f\|_{H^1_L,\text{max}} = \|M_L f\|_{L^1(\mathbb{R}^d)}$.

The purpose of this talk is to present that certain classes of Schrödinger operators admit special atomic decompositions for their Hardy spaces. Atoms that occur in these atomic decompositions are similar to that of the classical theory of real (local or global) Hardy spaces and their properties depend on $V$.

The results are joint works with Marcin Preisner and Jacek Zienkiewicz.
Generalized local Tb Theorems for Square functions
Ana Grau de la Herran
University of Helsinki, Finland

The local Tb Theorem is an $L^2$ boundedness criterion where the question about $L^2$ boundedness of an operator is reduced to verify its behavior to the local behavior of the operator on some test functions.

These theorems has been developed for Singular Integrals and Square functions with standard Calderón-Zygmund kernels in different settings by many authors, e.g. Auscher, Christ, Hofmann, Hytönen, Lacey, Martikainen, McIntosh, Mourgoglou, Muscalu, Nazarov, Qi Xiang Yang, Routin, Semmes, Tan, Tao, Tchamitchian, Thiele, Treil, Vahakangas, Volberg and Yan.

During the talk, I will present a local Tb Theorem for Square functions with non-pointwise bounded kernels.

The work presented is a joint work with prof. Steve Hofmann.

Strong $A_{\infty}$-weights are $A_{\infty}$-weights on metric spaces
Riikka Korte
University of Helsinki, Finland

We prove that every strong $A_{\infty}$-weight is a Muckenhoupt weight in Ahlfors-regular metric measure spaces that support a Poincare inequality. We also explore the relations between various definitions for $A_{\infty}$-weights in this setting, since some of these characterizations are needed in the proof of the main result. This is joint work with Outi Elina Kansanen.

On estimates of Calderon-Zygmund operators by dyadic positive operators
Andrei Lerner
Bar-Ilan University, Israel

In this talk I will discuss a recent result establishing that for any Banach function space $X$, the $X$-norm of any Calderon-Zygmund operator is controlled by the $X$-norm of a rather simple dyadic positive operator. In the particular case when $X = L^p(w)$ this result implies the $A_2$ conjecture and the two-weight bump conjecture.
Non-homogeneous $T_1$ theorem for bi-parameter singular integrals
Henri Martikainen
University of Helsinki, Finland

We prove a non-homogeneous $T_1$ theorem for certain bi-parameter singular integral operators. Moreover, we discuss the related non-homogenous Journé’s lemma and product BMO theory. (Joint work with Tuomas Hytönen.)

Singular integrals and removability for Lipschitz harmonic functions in Heisenberg groups
Pertti Mattila
University of Helsinki, Finland

The talk is based on joint work with V. Chousionis. We consider singular integrals on small (that is, measure zero and lower than full dimensional) subsets of metric groups. The main examples of the groups we have in mind are Euclidean spaces and Heisenberg groups. In Heisenberg groups we give some applications to harmonic (in the Heisenberg sense) functions analogous to some results known earlier in Euclidean spaces.

Discrete analogues in harmonic analysis
Mariusz Mirek
University of Wroclaw, Poland

I would like to present some recent results in discrete harmonic analysis and its connections with ergodic theory. Namely, we will consider $l^p$ – boundedness of maximal functions modeled on various subsets of $\mathbb{Z}^d$ and their applications to pointwise ergodic theorems.
**Sharp two weight estimates for Singular Integral Operators**
Carlos Pérez  
University of Seville, Spain

In this talk we will present some recent results concerning maximal Calderón-Zygmund singular integral operator $T^*$ and weights assuming no condition on the weight. To be more precise we will discuss the following fully optimal results

$$
\| T^* f \|_{L^p(w)} \leq c_T p' \left( \frac{1}{\delta} \right)^{1/p'} \| f \|_{L^p(M_{L(\log L)^{p-1+\delta}}(w))} \quad p > 1, w \geq 0, \delta > 0.
$$

and as consequence

$$
\| T^* f \|_{L^{1,\infty}(w)} \leq c_T \frac{1}{\delta} \int_{\mathbb{R}^n} |f(x)| M_{L(\log L)^{\delta}}(w)(x) \, dx \quad \delta > 0, w \geq 0
$$

As a consequence of this estimate we will discuss a quantitative version of the so called two weight bump $L\log L$ bump condition as well as a mixed $A_1 - A_\infty$ estimate.

These results are part of a joint work with T. Hytönen.

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**On Toeplitz products on Bergman space and two-weighted inequalities for the Bergman projection**
Sandra Pott  
Lund University, Sweden

In the early 90’s, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. The Hardy space case attracted much attention because of its close relation to the joint $A_2$ conjecture for the famous two-weight problem for the Hilbert transform in Real Analysis, but both conjectures, the Sarason conjecture for Toeplitz products on Hardy space and the joint $A_2$ conjecture, were shown to be false by F. Nazarov around 2000. Alternative, more complicated characterizations of boundedness of the two-weighted Hilbert transform by means of test functions have been the subject of extensive recent research by many authors, e.g. Lacey, Nazarov, Sawyer, Shen, Treil, Uriarte-Tuero, and Volberg.

The Bergman space case of Sarason’s conjecture is likewise connected to two-weighted inequalities for the Bergman projection.

In the talk, I will present a dyadic model for Toeplitz products on Bergman space, give necessary and sufficient conditions in terms of test functions in this case, and also comment on necessary and sufficient conditions for the Toeplitz products.

This is joint work with Alexandru Aleman and Maria Carmen Reguera (both Lund).
**Sharp Békolle estimates for the Bergman projection**

Maria Carmen Reguera  
University of Lund, Sweden

We look for sharp weighted estimates for the Bergman projection on the disc $\mathbb{D}$. The class of weights on $\mathbb{D}$ for which the Bergman projection $P$ extends to a bounded linear operator on weighted spaces $L^p(w, \mathbb{D})$ is the so called Békolle-Bonami class and it is usually denoted by $B_p$. In this talk, we find the linear bound for the Bergman projection in terms of the $B_2$ constant. The proof will consider a novel dyadic model for the Bergman projection. This is joint work with Sandra Pott.

**Layer potentials beyond singular integral operators**

Andreas Rosén  
Linköpings universitet, Sweden

In this talk I will show how the double layer potential and the gradient of the single layer potential operator for a divergence form elliptic equation can be constructed via functional calculus from an underlaying first order differential operator, for which Kato square root estimates hold. This recovers $L^2$ boundedness results, in the case of a single real divergence form equation, by Alfonseca-Auscher-Axelsson-Hofmann-Kim and Hofmann-Kenig-Mayboroda-Pipher. It also proves $L^2$ boundedness of the layer potentials for any second order divergence form system of equations.

**A basis for Dirichlet-Hardy spaces $\mathcal{H}^p$**

Eero Saksman  
University of Helsinki, Finland
Local entropy averages and the local distribution of measures.
Pablo Shmerkin
University of Surrey, United Kingdom

A central problem in geometric measure theory is to understand the relationship between the dimensions of sets and measures on one hand, and their local distribution on the other (expressed for example in terms of conical densities or porosity). Local entropy averages were recently introduced as a tool to attack this general problem, leading to a unified approach that also improved many previous results (and yielded a proof of a statement that had been claimed earlier, with an incorrect proof, by D. Beliaev and S. Smirnov). The method is dyadic in nature, and a random translation argument is often used to pass between Euclidean and dyadic versions of the problem. This is joint work with T. Sahlsten (Helsinki) and V. Suomala (Oulu).

Traces and embeddings of anisotropic function spaces with weights
Mark Veraar
Technische Universiteit Delft, Netherlands

In the talk we explain some new Sobolev embedding result for function spaces with weights. We show how these results can be used to characterize the trace spaces of several types of function spaces. In particular, we will obtain traces of intersected spaces which often arise in evolution equations. The talk is based on joint work with Martin Meyries.

A new characterization of Sobolev spaces in $\mathbb{R}^n$
Joan Verdera
Universitat Autònoma de Barcelona, Catalonia

I will describe a characterization of Sobolev spaces of any order of smoothness in $\mathbb{R}^n$, which is of a metric measure space nature. Thus Sobolev spaces of any order of smoothness can be defined in any metric measure space.