Introduction to LaTeX
Exercise Sheet 2 (Group 5)

Clifford Gilmore

20th March 2012

The `.tex` file containing your solutions to this exercise sheet should be emailed to `anni.luhtaniemi@helsinki.fi` before 15:00 on 28th March. The produced document should contain enough text to fill two pages. If you can't think of anything to write then you can find random text from Lorem Ipsum at http://www.lipsum.com/

The file name should be of the form `SurnameExercise2.tex`, e.g. `GilmoreExercise2.tex`.

Note, in the questions where you define a `newtheorem` environment, it is not necessary to match the theorem number in your solution with that of the question since it is automatically generated by LaTeX.

1. Create a document titled, *LaTeX Solutions 2*, with you as the author.
2. Create a section titled *Miscellaneous Mathematics* and reproduce the following text in this section (it does not have to be in an enumerated list):

   (a) Pythagoras states for a right angled triangle with side lengths a, b, c, then $a^2 + b^2 = c^2$.

   (b) Euler's identity states that

   $$e^{i\pi} + 1 = 0$$

   (c)

   $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k^2} = \frac{\pi^2}{6} \tag{1}$$

   (d) Pascal's rule is

   $$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \tag{2}$$

(you will need the `amsmath` package for this)

(e)

$$\int_0^{\frac{\pi}{2}} \cos x \mathrm{d}x$$

3. Create a new section titled, *Real and Fourier Analysis.* Define your own theorem environment with the `newtheorem` command and then use it to state the below theorem.

   Let $K = \{(x,y) \in \mathbb{R} : 0 \le x \le 1 \text{ and } 0 \le y \le x\}$ and let $f : K \to \mathbb{R}$ be an integrable function. Then

$$\int_K f dm_2 = \int_{[0,1]} \left(\int_{[0,x]} f(x,y) dm_1(y) \right) dm_1(x) = \int_{[0,1]} \left(\int_{[y,1]} f(x,y) dm_1(x) \right) dm_1(y)$$

4. Using the `proof` environment accessed through the `amsthm` package, add a proof for the above theorem. (You don't need to give the real proof, any paragraph of text will do)

5. Add a subsection called *Fourier Analysis* to the section and reproduce the below text in your document. (Hint: create a *Question* environment with the `newtheorem` command and use the `eqnarray` environment to align the calculations)

   Question 3.1. Let $f(\theta) = |\theta|, \theta \in [-\pi, \pi]$. Prove that $\hat{f}(0) = \frac{\pi}{2}$ and

$$\hat{f}(n) = \frac{-1 + (-1)^n}{\pi n^2}, \qquad n \neq 0.$$

   Proof. It is easily seen that

$$\hat{f}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| \ \mathrm{d}\theta = \frac{1}{\pi} \int_0^{\pi} \theta \ \mathrm{d}\theta = \frac{\pi}{2}.$$

   Next, for non-zero integers n we have,

$$\begin{aligned}
\hat{f}(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| e^{-in\theta} \ \mathrm{d}\theta \\
&= \frac{1}{\pi} \int_0^{\pi} \theta \cos n\theta \ \mathrm{d}\theta \\
&= \frac{1}{\pi} \left(\left[\frac{\theta \sin n\theta}{n} \right]_0^{\pi} - \frac{1}{n} \int_0^{\pi} \sin n\theta \ \mathrm{d}\theta \right) \\
&= -\frac{1}{n\pi} \left[-\frac{\cos n\theta}{n} \right]_0^{\pi} \\
&= \frac{\cos n\pi - 1}{\pi n^2} \\
&= \frac{(-1)^n - 1}{\pi n^2} = \begin{cases} 0 & \text{if } 2 \mid n, \\ -\frac{2}{\pi n^2} & \text{if } 2 \nmid n. \end{cases}
\end{aligned}$$

□

6. Add a new section titled *Linear Dynamics* and add the following theorem.

   Theorem 5.1 (Hypercyclicity Criterion)**.** Consider a separable Fréchet space X and an operator $T : X \to X$. If there exist dense subsets X_0, $Y_0 \subset X$, an increasing sequence $(n_k)_k$ of positive integers and maps $S_{n_k} : Y_0 \to X$, $k \geq 1$, such that for any $x \in X_0$, $y \in Y_0$

   (i) $T^{n_k} x \to 0$

   (ii) $S_{n_k} y \to 0$

   (iii) $T^{n_k} S_{n_k} y = y$

   then T is weakly mixing and in particular hypercyclic.