

MATHEMATICAL MODELING

EXAM 10-12-2009

Question 1

A predator, after it has found the prey, first has to fight with the prey. This fight is over when the prey escapes or when the prey is killed by the predator. In the first case, the predator takes a short rest before it searches for other prey again. In the second case, the predator eats the prey and takes a longer rest before it resumes hunting again. Ignore birth and death in the prey and in the predator.

Describe the above system with a network of unimolecular and bimolecular reactions and also give the corresponding population equations.

Question 2

When two individuals meet they may start a fight (or some other more playful interaction) during which they run around as a pair in a random, undirected manner. The interaction lasts for an exponentially distributed amount of time before they separate again. In order to avoid painful collisions, single individuals try to avoid the fast moving pairs by moving out of their way.

Model the above situation with a system of partial differential equations and reflecting boundary conditions.

Question 3

Consider the following epidemiological model:

$$\frac{dS}{dt} = \alpha S - \beta SI + \gamma I - \delta S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - (\delta + \varepsilon)I$$

with positive parameters and in particular with $\alpha > \delta$.

(a) What kind of processes may be described by the various terms of the above system? Give the corresponding reaction network.

(b) Give a phase plane analysis of the system and determine the stability each equilibrium. Use local stability analysis to determine stability if necessary.

(c) Show that cycles are not possible. Hint: use the Dulac function $u(S, I) = \frac{1}{SI}$.

Question 4

Consider the following resource-consumer system:

$$\partial_t R = 1 - \alpha R - \beta RC$$

$$\partial_t C = \gamma \beta RC - \delta C + D \partial_x^2 C$$

with positive parameters and in particular with $\alpha\delta < \gamma\beta$.

(a) What kind of processes are described by various terms of the above system? Give the corresponding reaction network.

(b) Reduce the system to a single equation for the consumer (C) assuming that the dynamics of the resource (R) is fast compared to that of the consumer.

(c) For the reduced system there exists a traveling wave solution $C(t, x) = c(x - vt)$ with $c(-\infty) = \frac{\beta\gamma - \alpha\delta}{\beta\delta}$ and $c(+\infty) = 0$ and wave speed $v \geq v_0 > 0$. Sketch the wave profile and calculate the minimum wave speed v_0 .