Project 3

(*pattern formation*)

When two individuals meet they may start a fight which involves a lot of random and undirected moving about. Single individuals avoid fighting pairs by moving out of their way. This can be modeled by the following PDE system:

$$\partial_t A = -\alpha A^2 + 2\beta B + \mu \partial_x \big(A \partial_x B\big) \quad (singles)$$

$$\partial_t B = \tfrac{1}{2}\alpha A^2 - \beta B + \nu \partial_x^2 B \qquad (pairs)$$

with reflecting boundaries at $x = 0$ and $x = 1$.

(a) Assuming fast reactions, calculate the quasi steady states of A and B as functions of the total (local) population density $C := A + 2B$, and use this to formulate a single PDE for C.

(b) Determine the stability conditions for the positive spatially homogeneous equilibrium solution of the PDE for C found in (a). In the case of instability, what pattern will emerge initially after a perturbation of the equilibrium?

(c) Use a phase-plane analysis to find out whether there are other (i.e., non-constant) positive equilibria? Under what conditions do they exist? What do they look like? Use a computer if needed.

(d) Solve the PDE for C numerically starting close to the equilibrium in (b) illustrating qualitatively different kinds of behavior.

Project 4

(*phase plane analysis*)

Consider the predator-prey system

$$\dot{x} = \alpha - \beta x - y f(x) \quad (prey)$$

$$\dot{y} = y f(x) - \delta y \qquad (predator)$$

with functional response

$$f(x) = \frac{x}{1 + x + x^2}.$$

(A humpbacked functional response may be the consequence of distraction of the predator when the prey density becomes too high.) Give a full phase plane analysis of the above system, distinguishing between various qualitatively different cases depending on the parameters α, β and δ. Use a computer if you wish.