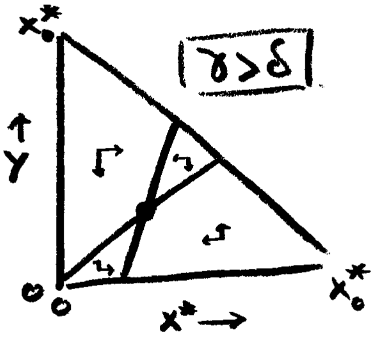
$\boxed{9\text{-}2\text{-}2012}$

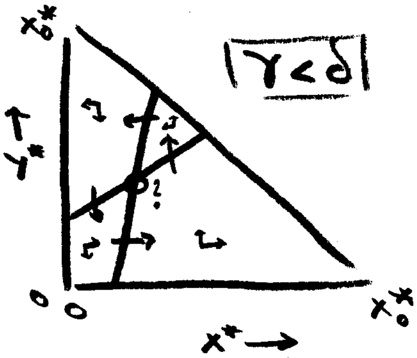
Remember the pred-prey example of $\boxed{02\text{-}02\text{-}2012}$, page 22 :

fast dyn
$$\begin{cases} \left(\dfrac{dn}{dt} = 0\right) & \text{(prey)} \\ \dfrac{dx^*}{dt} = -\alpha x^* n + \gamma y^* + \delta(x_0^* - x^* - y^*) & \text{(searching pred.)} \\ \dfrac{dy^*}{dt} = +\alpha x^* n - \gamma y^* - \beta y^* & \text{(pred-prey complex.)} \end{cases}$$

with phase portraits



stable equil.



stable or unstable. The picture does not tell.

To decide whether the equil in the case of $\boxed{\gamma < \delta}$ is stable or not we use "local stability analysis". (see Appendix. A)

$$\text{fast dyn.}\begin{cases}\dfrac{dx^*}{dt} = -\alpha x^* n + \gamma y^* + \delta(x_0^* - x^* - y^*) =: f_1(x^*, y^*)\\[2ex] \dfrac{dy^*}{dt} = +\alpha x^* n - \gamma y^* - \beta y^* \qquad\qquad =: f_2(x^*, y^*)\end{cases}$$

$$f(x^*, y^*) := \begin{pmatrix} f_1(x^*, y^*) \\ f_2(x^*, y^*) \end{pmatrix}$$

$$f(\bar{x}^*, \bar{y}^*) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{(equil.; see p. 23)}$$

$$\Rightarrow \quad f'(\bar{x}^*, \bar{y}^*) = \left(\begin{array}{c|c} -\alpha n - \delta & \gamma - \delta \\ \hline \alpha n & -\gamma - \beta \end{array}\right)$$

$$\Rightarrow \begin{cases} \det f'(\bar{x}^*, \bar{y}^*) = (\alpha n + \delta)(\gamma + \beta) \underbrace{- \alpha n(\gamma - \delta)}_{>0 \text{ for } \gamma < \delta} \\[1ex] \qquad > 0 \\[1ex] \operatorname{trace} f'(\bar{x}^*, \bar{y}^*) = -\alpha n - \delta - \gamma - \beta < 0. \end{cases}$$

$\xrightarrow{\text{(App. A)}}$ $(\bar{x}^*, \bar{y}^*)$ is stable.

	det	
	stab.	unstab.
		trace
	unstab.	unstab.

So, our derivation of the slow dyn. on p. 28 (2-2-2012):

$$\frac{dn}{dt^*} = -\frac{\alpha\beta\delta x_0^* n}{(\beta+\gamma)\delta + \alpha(\beta+\delta)n}$$

also holds for $\gamma < \delta$.