MATHEMATICAL MODELING 2012
EXERCISES 21-23

21.
Suggest a set of differential equations plus appropriate boundary conditions for each of the following situations:

(a) A predator-prey system in which the predator moves towards higher prey densities and the prey towards lower predator densities. Assume further that the movement of the individual prey and the individual predator also has a random (i.e., undirected) component.

(b) A population of cells produces a substance that diffuses and is gradually broken down at some given rate. The cells move towards higher concentrations of the substance but also have a random component to their movement.

(c) Oxygen diffuses from the surface of a vertical water column to the bottom where it is absorbed by a layer of debris. The absorption rate is proportional to the density of the debris. The oxygen that is being absorbed is used (by bacteria) to decompose the debris, the density of which therefore decreases in time.

(d) Same situation as in **(c)**, but now there also is a population of zoo-plankton. The movement of the individual plankton has a random component as well as a systematic component directed towards higher oxygen concentrations. The individuals reproduce and absorb oxygen at a rate proportional to the oxygen concentration. the per capita death rate is constant. Dead individuals sink to the bottom and contribute to the debris layer.

22.
What kind of processes might be described by the following equations?

(a)

$$\partial_t c = \partial_x (vc) - \delta c$$

(b)

$$\partial_t c = rc\left(1 - \frac{c}{K}\right) + \partial_x\Big(D(c)\partial_x c - vc\Big)$$

(c)

$$\partial_t c_i = \partial_x\Big(c_i \partial_x (c_1 + c_2)\Big) \quad (i = 1, 2)$$

23.
Find all possible traveling wave solutions and corresponding boundary conditions at $x = \pm\infty$ for the equation

$$\partial_t n = rn(1-n)(n-a)D\partial_x^2 n \quad (r, a > 0)$$