MATHEMATICAL MODELING 2012
EXERCISES 13-17

13. (a) Give the delay differential equation (DDE) for the population density if birth and death are modeled as monomolecular reactions and a newborn becomes reproductively active only after T time units. (b) Same question, but now instead of the developmental delay in the newborns there is a rest period of length T after having given birth.

14. (a) Give the DDEs for a prey-predator system assuming logistic growth of the prey population if predators are absent, and a functional response Holling-I for the predator, and a juvenile period of length T for the predator only, during which the individuals do not hunt and do not reproduce. (b) Give a single DDE for the predator population assuming (in addition to the above) that the prey dynamics are fast compared to that of the predator.

15. Give the DDE for the population density if birth and death are modeled as monomolecular reactions without developmental delays but assuming that no-one gets older than T time units. (*Hint:* first formulate the transport equation and the renewal equation for the age distribution, and next integrate the transport equation to get a DDE for the total population density N.)

16. Give the DDEs for the dynamics of a disease if the duration of the disease has a fixed length T (after which the individual recovers again) and death in susceptible (S) and in infected (I) individuals is modeled as a monomolecular reaction, and the population birth rate (giving rise to susceptible only) is a constant B. (Like in exercise 15, first formulate the transport equation and the renewal equation for the age distribution of the infected, and only then integrate the transport equation to get the DDEs for the total densities of the susceptible and the infected.)