Titles and abstracts

Denis Bernard

Ecole Normale Supérieure / CNRS

A detour into wave function collapses from Brownian motions

Jérémie Bouttier

CEA Saclay, Institut de physique théorique

The nested loop approach to the O(n) model on random maps

We consider the $O(n)$ loop model on random planar maps (i.e. graphs embedded in the sphere). We explain how an elementary combinatorial decomposition, which consists in cutting the maps along the outermost loops, allows to relate the $O(n)$ model to the simpler problem of counting maps with controlled face degrees. This translates into a functional relation for the "resolvent" of the model, which is exactly solvable in several interesting cases. We then look for critical points of the model: our construction shows that at the so-called non-generic critical points, the $O(n)$ model is related to the "stable" maps introduced by Le Gall and Miermont.

Luigi Cantini

Université Cergy-Pontoise

A one-parameter refinement of the Razumov-Stroganov correspondence

In 2001 Razumov and Stroganov conjectured that the (properly normalized) components of the ground state of the dense O(1) loop model on a semi-infinite cylinder enumerate fully-packed loop (FPL) configurations on the square, with alternating boundary conditions, refined according to the link pattern for the boundary points. This conjecture has arisen a lot of interest both in the physics and in the mathematics community. In this talk, after reviewing the main background, I shall discuss a stronger (but easier to prove!) version of the correspondence: on the 'dense O(1) side', the ground state of Hamiltonian H of the loop model is replaced by the one of the Scattering Matrix S(t); on the 'FPL side', one considers the refinement according to the position of the unique straight tile on the last row.

John Cardy

University of Oxford

Holomorphic parafermions on the lattice and in conformal field theory

Dmitry Chelkak

Steklov Institute and St. Petersburg State University

Conformal invariance of spin correlations in the planar Ising model

We rigorously prove existence and conformal covariance of scaling limits of spin correlations in the critical Ising model (defined on square grid approximations of a simply connected planar domain). This solves a number of conjectures coming from physical and mathematical literatures. The proof is based on convergence results for discrete holomorphic spinor observables which allow us to compute the logarithmic derivatives of those correlations with respect to positions of points, and relate the correlations for various boundary conditions to each other. Based on a joint work with Clement Hongler and Konstantin Izyurov (arXiv:1202.2838 and arXiv:1105.5709).

Béatrice de Tilière

University Pierre et Marie Curie - Paris 6

Mappings of the 2-dimensionl Ising model

We shall present two mappings of two independent Ising models on a graph. The first one holds at criticality and is a mapping to critical trees. The second one is done in collaboration with Cdric Boutillier. It is a mapping of XOR-loops to loops of quadri-tilings, a bipartite dimer model.

Hugo Duminil-Copin

University of Geneva

Crossing probabilities in the study of critical lattice models

We will describe several theorems dealing with crossing probabilities for rectangles in critical percolation, Ising and random cluster models. These theorems provide lower and upper bounds for crossing probabilities in divers context. We will discuss the relevance of boundary conditions, of the shape of domains for which bounds are available, etc. In particular, we will explain which applications follow from theorems dealing with specific boundary conditions only (such as wired, free, periodic, free/wired/free/wired) and which applications require results uniform in boundary conditions. At the end, we will describe why extending bounds on crossing probabilities to arbitrary topological rectangles is crucial to the study of arm-exponents and scaling limits in the case of the Ising model.

This talk is based partly on joint work with P. Nolin, C. Hongler and D. Chelkak.

Bertrand Duplantier

CEA Saclay, Institut de physique théorique

SLE, KPZ and Liouville Quantum Gravity

When two boundary arcs of a Liouville quantum gravity random surface are conformally welded to each other (in a boundary quantum-length-preserving way) the resulting interface is a random curve described by the Schramm-Loewner evolution (SLE). This allows to develop a theory of

quantum fractal measures (consistent with the Knizhnik-Polyakov-Zamolochikov relation) and to analyze their evolution under conformal welding maps related to SLE. As an application, one can construct quantum length and boundary intersection measures on the SLE curve itself. (Joint work with Scott Sheffield, MIT.)

Paul Fendley

University of Virginia

Discrete holomorphicity from topology

Two-dimensional critical integrable models have long been known to be intimately connected to three-dimensional topology. For example, knot and link invariants such as the Jones polynomial can be obtained as a certain limit of Boltzmann weights satisfying the (non-linear) Yang-Baxter equation. Recently, Cardy and collaborators have shown that such Boltzmann weights often can be obtained from a set of linear relations ensuring that certain operators are discretely holomorphic. I explain how discrete holomorphicity naturally arises by defining these operators in terms of three-dimensional link invariants. This allows the results of Smirnov and Cardy et al to be extended both to lattice height models, and to new hierarchies of integrable models (e.g. those associated with the Birman-Wenzl-Murakami algebras). It also provides a direct connection between integrability and conformal field theory.

Emmanuel Guitter

CEA Saclay, Institut de physique théorique

Distance statistics in random maps

I will review a number of exact results for the statistics of distances in random maps. These include explicit formulas for distance-dependent one-, two-, and three-point functions in various geometries (planar maps, toroidal maps, maps with a boundary) both at the discrete level and in the continuous scaling limit of large random maps. I will also discuss the bijective method that led to these results.

Anthony Guttmann

University of Melbourne

A generalised identity for self-avoiding walks on the honeycomb lattice

A recently proved identity by Duminil-Copin and Smirnov connecting certain SAW generating functions at their radii of convergence is extended in three ways. Firstly by including surface interactions, allowing exact values for critical surface fugacities to be proved. Secondly, by varying the geometry, giving rise to critical exponent inequalities, and thirdly, by generalising the identity away from the critical temperature, thereby allowing exponent relations connecting the winding distribution to surface critical exponents.

Christian Hagendorf

University of Geneva

Spin chains and dynamical lattice supersymmetry

The topic of this talk is the connection between spin chains/vertex models and dynamical supersymmetry on the lattice. The corresponding supercharges in the lattice models act non-locally and change the number of sites. The simplest example is the spin-1/2 XXZ chain/six-vertex model with periodic boundary conditions at the combinatorial point, which possesses an N=(2,2) supersymmetry on the lattice. I will explain the representations of the lattice supersymmetry algebra for models with arbitrary spin, and show how to construct the lattice equivalents of the minimal series of N=(2,2) superconformal theories. Furthermore I will discuss in detail the Fateev-Zamolodchikov spin chain, and present some evidence that its ground states, the supersymmetry singlets, are related to the weighted enumeration of alternating sign matrices.

Clément Hongler

Columbia University

Conformal Invariance of Ising Interfaces

We consider the interfaces that arise in the critical Ising model in various setups. We will discuss their scaling limits in terms of SLE processes and variants, and also the recent tools introduced to investigate these questions (crossing probabilities and martingale observables).

Yacine Ikhlef

University of Geneva

Discretely holomorphic parafermions and integrable boundary conditions

In two-dimensional statistical models possessing a discretely holomorphic parafermion, we introduce a modified discrete Cauchy-Riemann equation on the boundary of the domain, and we show that the solution of this equation yields integrable boundary Boltzmann weights. This approach is applied to (i) the square-lattice O(n) loop model, where the exact locations of the special and ordinary transitions are recovered, and (ii) the Fateev-Zamolodchikov Z_N spin model, where a new rotation-invariant, integrable boundary condition is discovered for generic N.

Konstantin Izyurov

St. Petersburg State University, Chebyshev Laboratory

Ising interfaces and SLE(3) in multiply connected domains

We prove a general theorem on convergence of multiple Ising interfaces in finitely connected domains to SLE(3) variants. Explicit examples include multiple SLE(3) in simply-connected domains, corresponding to alternating +/- boundary conditions, radial SLE(3) arising in the Ising model with a disorder insertion, and annulus SLE(3). In general, the limiting law on curves can be

described in terms of scaling limits of corresponding Ising partition functions, or in terms of solutions to a Riemann boundary value problem.

Jesper Jacobsen

Ecole Normale Supérieure

Exact critical manifolds from graph polynomials

Any two-dimensional infinite regular lattice G can be produced by tiling the plane with a finite subgraph B of G; we call B a basis of G. We introduce a two-parameter graph polynomial $P_B(q, v)$ that depends on B and its embedding in G. The algebraic curve $P_B(q, v) = 0$ is shown to provide an approximation to the critical manifold of the q-state Potts model, with coupling $v = exp(K) - 1$, defined on G. For larger bases B the approximations become increasingly accurate, and we conjecture that $P_B(q, v) = 0$ provides the exact critical manifold in the limit of infinite B. Furthermore, for some lattices G, or for the Ising model ($q = 2$) on any G, $P_B(q, v)$ factorises for any choice of B: the zero set of the recurrent factor then provides the exact critical manifold. In this sense, the computation of $P_B(q, v)$ can be used to detect exact solvability of the Potts model on G. We have also obtained similar results for site percolation.

Kurt Johansson

KTH Royal Institute of Technology

Continuum scaling limits in random tilings

Scaling limits close to the boundary between a frozen and liquid region in random tiling models of certain planar regions give rise to limiting processes that also arise in random matrix theory. The basic limiting process is the Airy process which also occurs in random growth models. I will discuss this example and also a new scaling limit called the tacnode.

Richard Kenyon

Brown University

Conformal Invariance of double-dimer paths

Using an extension to quaternionic variables of the standard methods of Kasteleyn, we prove the conformal invariance of the double-dimer loops on the square grid. Similar methods apply to the loops in the "cycle-rooted spanning tree" model.

Ivan Kostov

CEA Saclay, Institut de physique théorique

O(n) loops in complex magnetic field

The $O(n)$ loop model on planar graphs is solved in presence of constant magnetic field H. In this case the geometrical expansion involves (self and mutually avoiding) loops with fugacity n and

open lines with fugacity H^2. An equation for the universal part of the specific free energy is derived using the correspondence with a matrix model. The specific free energy has a pair of Yang-Lee edges on the high-temperature sheet and a Langer type branch cut on the low-temperature sheet at $H = 0$. The exponent of the Langer type singularity is compatible with a conjecture by A. and Al. Zamolodchikov about the decay rate of the metastable vacuum in presence of Liouville gravity.

Gregory Lawler

University of Chicago

Recent Results on SLE

I will discuss a number of recent results about SLE paths that I have obtained in collaboration with a number of collaborators. The main themes will be natural parametrization and SLE in multiply connected domains.

Zhongyang Li

University of Cambridge

Critical Temperature of Periodic Ising Models

A periodic Ising model has interactions which are invariant under translations of a full-rank sublattice $\mathfrak{L}$ of $\mathbb{Z}^2$. We prove an exact, quantitative characterization of the critical temperature, defined as the supremum of temperatures for which the spontaneous magnetization is strictly positive. For the ferromagnetic model, the critical temperature is the solution of a certain algebraic equation, resulting from the condition that the spectral curve of the corresponding dimer model on the Fisher graph has a real zero on the unit torus. With our technique we provide a simple proof for the exponential decay of spin-spin correlations above the critical temperature, as well as the exponential decay of the edge-edge correlations for all non-critical edge weights of the corresponding dimer model on periodic Fisher graphs.

Ioan Manolescu

University of Cambridge

Bond Percolation on Isoradial Graphs

The star-triangle transformation is used to obtain an equivalence extending over a set bond percolation models on isoradial graphs. Amongst the consequences are box-crossing (RSW) inequalities and the universality of alternating arms exponents (assuming they exist) for such models, under some conditions. In particular this implies criticality for these models. The work is joint with Geoffrey Grimmett.

Pekka Nieminen

University of Helsinki

Gaussian free field and Hadamard's variational formula

We revisit the classical variational formula due to Hadamard and interpret it in terms of a natural integral operator. We use this operator to provide a construction of the Gaussian free field on a planar domain. Joint work with Haakan Hedenmalm (KTH, Stockholm).

Bernard Nienhuis

University of Amsterdam

Elliptic Bethe Ansatz for itinerant fermions on a chain

Recently Fendley and Schoutens presented a simple construction of a super-symmetric (SUSY) Hamiltonian for interacting fermions on a lattice. A simple one-dimensional example is equivalent to the XXZ spin chain, and can be solved with the Bethe Ansatz. The construction of the SUSY is sufficiently general that it allows for spatial variation of the parameters. We show that with periodic spatial variation with period three, the model still admits a Bethe Ansatz, now with an elliptic parametrization.

Eero Saksman

University of Helsinki

An application of stochastic games to PDE:s

We give a simple proof of Harnack's inequality for p-harmonic functions via stochastic games. The talk is based on joint work with Hannes Luiro and Mikko Parviainen (University of Jyväskylä)

Hubert Saleur

CEA Saclay, Institut de physique théorique

Edge states in the spin quantum Hall effect

The absolute (that is, not modulo 2π) value of the topological angle in conformal sigma models affects the properties of the boundary conformal field theory. It corresponds physically to the presence of extra edge states. This problem in class C (spin quantum Hall) can be mapped on to a geometrical problem which is like percolation in the bulk, but with extra life on the boundary. Transport properties are modified in consequence, and non trivial (in fact, irrational) new exponents obtained. These can be observed in simulations on Chalker Coddington like network models, and hopefully experimentally some day. In this talk, I will discuss both the physics underlying the problem, and the technical aspects underlying its solution.

Jacob Simmons

Maine Maritime Academy

Six-point CFT correlation functions with applications to critical loop models

Well discuss correlation functions in conformal field theories with applications to critical loop models. Our major result is a chiral six-point function that gives the density of clusters in the presence of a 4SLE process, or alternately corresponds to a set of bulk parity operators near a boundary with dipolar and fixed boundary conditions. Well explicitly describe applications in percolation and self-avoiding loop models, both theories with zero central charge, placing emphasis on the logarithmic aspects of these theories.

Christian Webb

University of Helsinki

An overview of recent results concerning low temperature Gibbs measures for logartihmically correlated Gaussian fields

We give a brief overview of some current results concerning the low temperature behavior of Gibbs measures for which the Hamiltonian is a Gaussian field with logarithmic correlations on a discretization of $\mathbb{R}^n$.
We discuss recent results by Madaule; Aidekon and Shi; Webb; Barral, Rhodes and Vargas which imply that for a hierarchical field (i.e. the case of multiplicative cascades) even below the critical temperature the Gibbs measures have a continuum limit and can be described explicitly.
We also discuss work by Arguin and Zindy concerning the replica overlap for a certain Gaussian field with translation invariant logarithmic correlations. Their results suggest that the atomic structure of the low temperature Gibbs measures for models with Hamiltonians similar to the Gaussian free field should be very similar to the hierarchical case.
Finally we discuss work by Bolthausen, Deuschel and Zeitouni and Bramson and Zeitouni about the tightness of the maximum of the discrete two-dimensional Gaussian free field. Their results can be seen as the first step in proving existence of a continuum limit of the low temperature Gibbs measure and they suggest similar behavior as in the hierarchical case.

Paul Zinn-Justin

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