Exercises II

1. Use the sample of opinions about the height of Eiffel collected from this course. Take the minimum and maximum value of each opinion, and make a uniform density, $\mathrm{U}(\min_i, \max_i)$, from each, $i = 1, \ldots, n$. Combine them into a single probability density by using the product method and the method of sum. Write the density function and *sketch* its features graphically in each case. The height of Eiffel is 324m (with antenna). The listed heights of other 'comparable' monuments may have affected the opinions? The **mi**nimum and **ma**ximum values are listed here pairwise:

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mi=c(100,240, 60,140,150,250,250,200,100,180, 70, 70,100,300,290,150,100,300,190)
ma=c(300,320,200,450,250,450,350,600,300,300,150,130,500,400,310,500,400,400,250)
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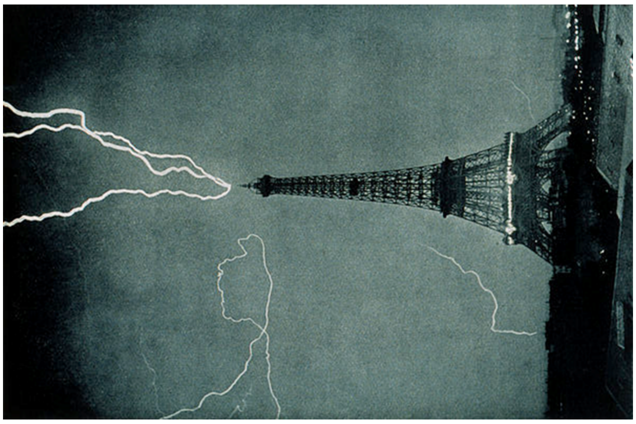

Figure 1: What's the height of Eiffel? Näsinneula, Tampere 1971: 168m. Tallinn TV Tower 1980: 312m. Petronas Tower, Kuala Lumpur, 1998: 452m. Sears Tower, Chicago, 1974: 442m. Empire State Building, New York, 1931: 381m. WTC towers, New York, 1972: 417m.

2. Let $\pi(X \mid N, p) = \mathrm{Bin}(N, p)$ and $\pi(p) = \mathrm{Beta}(\alpha, \beta)$. Calculate analytically from the Bayes formula that the posterior distribution $\pi(p \mid X)$ is $\mathrm{Beta}(X + \alpha, N - X + \beta)$.

3. Use the sample data from this course: number of left-handed $X = 2$ / $N = 15$ (males) and $X = 0$ / $N = 3$ (females). Find the posterior density of the percentage of left-handed in the population of female and male students. Assume 'uninformative' prior. Is it really uninformative, considering what you really knew before the sample already? Summarize the results. According to literature, (Tiede-lehti, Google...), about 5-20% of the population is thought to be left-handed. Formulate a prior density reflecting this evidence, and recalculate the posteriors. Is there a difference? Judge the weight of the prior information against the weight of the sample.

4. According to birth statistics, there were 319,157 boys and 306,376 girls born during 1990-1999 in Finland. As Laplace, try to analyze the percentage θ of boys born in a large number of births, based on this evidence, by using paper and pencil. You need to approximate the beta-density posterior by e.g. normal density by matching the relevant parameters. Try to calculate $P(\theta > 0.5 \mid \text{data})$. You may use tabulated values (e.g. from some software) for cumulative normal distribution.

5. Consider the binomial model $P(X \mid N, p) = \mathrm{Bin}(N, p)$ and the posterior density for p, given X. This could result from an experiment where we decide to collect N samples and then observe X of them to be positive. Show that the posterior density is the same for the following experiment: a population has a fraction p of positives, and we continue testing until we obtain X positives. Why is this so?