EVOLUTION AND THE THEORY OF GAMES

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Exercises 13-12-2011

20. Consider the Iterated Stag Hunt: Each day, two players go out to hunt a stag (=male deer). They lay in ambush, waiting for the stag to come nearer. The first animal to come within reach, however, is a hare. The options for the players are: (S) ignore the hare and wait for the stag or (H) go for the hare and spoil the trap for the stag. If both players wait for the stag, they share the prize; if both players go for the hare they share the (smaller) prize; if only one of the players goes for the hare, he gets the hare with only half the probability, but he will not share the prize, and the other player will get nothing.

It takes two hunters to kill a stag, but one hunter can capture a hare. Moreover, the stag can defend itself, but the hare cannot. Killing the stag, therefore, is riskier for the hunters than killing the hare. The probability that both players are fit enough to go for a hunt the next day again therefore depends on the strategies.

Γ	S	H
S	$R+\delta\,\Gamma$	$\Delta\Gamma$
H	$r+\Delta\Gamma$	$r+\Delta\Gamma$

(payoffs to row-player)

where R is half the expected value of the stag and r half the expected value of the hare, and where δ and Δ are the probabilities of a next round.

(a) Analyze the iterated Stag Hunt with $0 < r < R$ and $0 < \delta < \Delta < 1$.

Suppose that if a stag is killed, the hunters can afford to skip one day of hunting and instead they spend at home. The probability of surviving a day at home is the same as for a day of hunting hare. If a hare is killed, the hunters have to go out the next day again. This modification of the game can be modeled with the following two sub-games:

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Γ_1	S	H
S	$R + \delta\, \Gamma_2$	$\Delta\Gamma_1$
H	$r + \Delta\Gamma_1$	$r + \Delta\Gamma_1$

(payoffs to row-player)

Γ_2	rest
rest	$\Delta\, \Gamma_1$

(payoffs to row-player)

(b) Analyze the game $\Gamma = (\Gamma_1, \Gamma_2)$ with $0 < r < R$ and $0 < \delta < \Delta < 1$.

Consider the Stag Hunt with three strategies: "every day a stag" (S) and "every day a hare" (H) and "one rest day after a day of successful stag hunting" (SR). If S goes with SR, then the latter takes a day off, while the former goes alone hunting a hare. This situation can be modeled by the three-stage game $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$

and the three sub-games:

Γ_1	S	SR	H
S	$R + \delta\, \Gamma_1$	$R + \delta\, \Gamma_2$	$\Delta\Gamma_1$
SR	$R + \delta\Gamma_2$	$R + \delta\, \Gamma_3$	$\Delta\Gamma_1$
H	$r + \Delta\Gamma_1$	$r + \Delta\Gamma_1$	$r + \Delta\Gamma_1$

(payoffs to row-player)

Γ_2	SR-player
S-player	$r + \Delta\, \Gamma_1\ ,\ \Delta\, \Gamma_1$

(asymmatric game)

Γ_3	SR-player
SR-player	$\Delta\, \Gamma_1\ ,\ \Delta\, \Gamma_1$

(payoff to row-player)

Notice that although Γ_2 is an asymmetric game, the players are given only one option: "rest" for the SR-player and "hunt a hare" for the S-player.

(c) Analyze the game $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ with $0 < r < R$ and $0 < \delta < \Delta < 1$.

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21. Consider the following resident dynamics for different prey strategies and a single non-evolving predator:

$$\text{prey}: \quad \frac{\dot{n}_i}{n_i} = r(x_i)\left(1 - \frac{\sum_{j=1}^{k} n_j}{K}\right) - \frac{\beta(x_i)p}{1+\sum_{j=1}^{k}\beta(x_j)T(x_j)n_j} \quad i = 1, \dots, k$$

$$\text{pred}: \quad \frac{\dot{p}}{p} = \frac{\sum_{j=1}^{k}\gamma(x_j)\beta(x_j)n_j}{1+\sum_{j=1}^{k}\beta(x_j)T(x_j)n_j} - \delta$$

(a) Show that coexistence of different prey strategies is not robust.

(b) Write down the expression for the invasion fitness of a prey mutant with strategy y.

(c) Show that strategy y can invade a monomorphic resident population of strategy x if $\beta(y)/r(y) < \beta(x)/r(x)$.

(d) Do the dynamics of the predator really matter?