CONFORMAL FIELD THEORY, EXERCISE SET 9

1. Derive the operator product expansion for $\psi(z)\psi(w)$ in the case of the fermionic field $\psi(z) = \sum_{n\in\mathbb{Z}+1/2} a_n z^{-n-1/2}$ from the anticommutation relations

$$a_n a_m + a_m a_n = \delta_{n+m}.$$

2. Derive the operator product expansion for $\phi(z)T(w)$ when $T(w)$ is the Virasoro field and $\phi(z) = \sum_{n\in\mathbb{Z}} a_n z^{-n-1}$ is the free bosonic field, with commutation relations $[a_n, a_m] = n\delta_{n+m}$ and $[L_n, a_m] = -m a_{n+m}$.

3. Assume that $A(z)$ and $B(z)$ are fields acting in a vector space V. Show that the normal ordered product $: A(z)B(w) : v$ makes sense as a formal distribution in z (with values in V) even at $w = z$ for all $v \in V$. Show that $: A(z)B(z) :$ is a field.

4. Let $\phi(z)$ be the free bosonic field of exercise 2. Then $L(z) =: \phi(z)\phi(z) :$ is also a field. Compute the commutation relations $[L(z), L(w)]$.

5. Let $\psi(z)$ be the fermionic field of exercise 1. Set now $L(z) = \frac{1}{2} : (\partial\psi(z))\psi(z) :$ and compute the commutators $[L(z), L(w)]$.