CONFORMAL FIELD THEORY, EXERCISE SET 5

1. On the torus $T = S^1 \times S^1$ one can define the symplectic form $\lambda\, d\phi_1 \wedge d\phi_2$ (with $0 \le \phi_i \le 2\pi$) for any real number $\lambda \neq 0$. For which values of λ there is a complex line bundle L with curvature $F = i\omega$ and construct the line bundle for allowed values of λ.

2. Construct a hermitean connection in the line bundle L in Exercise 1 and the prequantization operators for the hamiltonian functions $\sin(\phi_i)$.

3. Compute the hamiltonian vector field X_α corresponding to the function $\ell_\alpha = -\frac{\pi}{k}\int_{S^1} \alpha(x) \mathrm{tr}\, \xi(x)^2 dx$ on the phase space of the WZW model.

4. Compute the components $T_{\mu\nu} = \frac{\delta L}{\delta g^{\mu\nu}}$ of the energy momentum tensor of the Wess-Zumino-Witten model at the background metric $g_{\mu\nu} =$ the Minkowski metric and check that indeed $T^\mu_\mu = 0$.

5. On the group Diff(S^1) one can define a constant coefficient 2-form ω in the following way. Since Diff(S^1) is a group we identify a tangent vector at arbitrary position $g \in$ Diff(S^1) as a tangent vector at the identity, that is, as an element of the Lie algebra of the group; in this case a Lie algebra element is a vector field $\alpha(x)\frac{d}{dx}$ on the circle. Define

$$\omega(\alpha, \beta) = \int_{S^1} \alpha'(x)\beta''(x)dx.$$

Since ω has constant coeffcients (does not depend on the variable $g \in$ Diff(S^1)) it is closed. Is it a symplectic form? If not, can you improve the situation to make it nondegenerate (compare the Exercise 5/4)? In any case, fix a complex line bundle over Diff(S^1) with curvature $F = i\omega$. Compute the commutators $[\nabla_\alpha, \nabla_\beta]$. Here ∇_α is the covariant differentiation in the direction of the left invariant vector field X_α on Diff(S^1) defined by the Lie algebra element $\alpha\frac{d}{dx}$.