CONFORMAL FIELD THEORY, EXERCISE SET 4

1. Derive the Euler-Lagrange field equations of the Wess-Zumino-Witten model from the WZW Lagrangian.

2. Using Darboux coordinates show that $[X_f, X_g] = X_{\{f,g\}}$ where X_f is he Hamiltonian vector field corresponding to a smooth function f.

3. Show that $\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0$.

4. Let $\Omega G = \{g \in LG | g(0) = g(2\pi) = 1\}$ be the *based loop group.* Identifying tangent vectors on ΩG at arbitrary point g as tangent vectors at the identity, that is, as elements of the Lie algebra $\Omega \mathfrak{g}$, through right the translation by g^{-1}, define the 2-form

$$\omega(X, Y) = \int_0^{2\pi} \operatorname{tr} X(x) Y'(x) dx.$$

Show that ω is a symplectic form on ΩG. The trace is taken in a faithful representation of G, e.g. the defining representation for $G = SU(n)$.

5. Show that also the modified 2-form

$$\omega(X, Y) = \int \operatorname{tr} (g' g^{-1} [X, Y] + X Y') dx$$

is nondegenerate on ΩG. Is ω a symplectic form?