University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 10, 21.11.2011

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. Suppose that we are given some data points $(x_k, y_k), k = 1, ..., m$, which seem to contain "two linear trends with a break point". For instance the data

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x=-2:0.1:4; y=0.2*sin(3*x);
y(x<1)=y(x<1)+0.5*(x(x<1)-1);
y(x>=1)=y(x>=1)+2*(x(x>=1)-1);
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has this feature with the break point $x = 1$. It would seem to be a natural idea to fit, instead of the LSQ line, "a line with a breakpoint"to the data.

Write a program that prompts the user to enter by a mouse click a point (s, t) in the plane (the simplest method is to use the built-in function ginput but also getpts.m or some modification of it could be used). Then fit a polygonal line with break point (s, t) to these points. That is, fit a line $y = k_1 x + b_1, x < s$, to the points $(x_k, y_k), x_k < s$, and a line $y = k_2 x + b_2, x > s$, to the points $(x_k, y_k), x_k > s$.

2. Suppose that the vertices a, b, c of a triangle are on the boundary of the square $[0, 1] \times [0, 1]$. Within this triangle we choose 1000 triples of random points (a_1, b_1, c_1) and for each such triple we compute the ratio of the area of the new triangle to the area of the triangle (a, b, c). What is the mean value of these ratios? Use the built-in function inpolygon to check that the points a_1, b_1, c_1 are within the original triangle.

3. The triangle inequality states that

$$d(x, y) \leq d(x, z) + d(z, y)$$

for all $x, y, z \in \mathbb{R}^2$ if $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$. Define now a new function

$$d_2(x, y) = |x_1 - y_1|^{1/2} + |x_2 - y_2|^{1/3}.$$

Carry out tests to verify whether this function satisfies the triangle inequality.

FILE: ~/mme/demo11/d10/d10.tex — 14. marraskuuta 2011 (klo 16.05).

4. We consider three methods of fitting a second degree polynomial to $\sin(\pi * x)$ on $(0,1)$.

(a) The first method is to use the program parfit. Let xdata=0:0.1:1; ydata=sin(pi*xdata); and carry out the fitting.

(b) The second method is to use the LSQ-fitting in the sense of the L^2 norm. In other words, we wish to find c_1, c_2, c_3 so that

$$f(c) = \int_0^1 (\sin(\pi * x) - c_1 x^2 - c_2 x - c_3)^2 dx$$

will be minimal. Set up the normal equations and write the resulting 3×3 linear system of equations. Use e626.m to implement this idea.

(c) The third method is to use the built-in function polyfit for fitting second degree polynomial.

5. The built-in function \ of MATLAB solves also systems of linear equations in the LSQ sense. For instance, if we have more linear equations than unknowns, we do not expect to have "an exact solution in the sense of linear algebra"but we still might have an LSQ-solution. Use this fact in the following two cases:

(a) Plot the lines $y = 0.2 - 0.5 * x$, $y = 0.8 - 2 * x$, $y = 0.2 + 2 * x$, $y = -0.3 + 1.5 * x$, $y = 0.0 + 0.95 * x$, Write these equations in the form $a[x; y] = b$ where a is a 5×2 matrix and solve the system in the LSQ sense for $[x; y]$ and plot this LSQ solution of the system in the same figure.

(b) The file d105.dat on the www-page contains 8 temperature measurements in the format $(x_i, y_i, g_i), i = 1, ..., 8$. We wish to fit the quadratic function

$$g(x, y) = ax^2 + by^2 + cxy + dx + ey + f$$

to this data. Write the corresponding linear system of equations and solve it for the unknown vector (a, b, c, d, e, f). Compute the value of the temperature at the location $(5, 5)$. For each data point (x_i, y_i), compute the difference $g_i - g(x_i, y_i)$. Graph the surface $z = g(x, y)$.

6. Let A, B be $m \times n$ and $p \times q$ matrices. Their Kronecker product $kron(A, B)$ (also called tensor or direct product) is the $mp \times nq$ matrix

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[a(1,1)B ... a(1,n)B; ...; a(m,1)B ... a(m,n)B ];
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For simplicity we write here $Kron(A, B) = [A, B]$. Verify experimentally the following properties with the help of MATLAB's built-in function kron and also state possible restrictions for the dimensions of the involved matrices:

(a) $[A,[B,C]] = [[A,B],C]$,

(b) $[A,B+C] = [A,B]+[A,C]$,

(c) $[A^{-1},B^{-1}] = [A,B]^{-1}$

(d) $[A^T,B^T] = [A,B]^T$

(e) If B and C have SVDs $B = U_1W_1V_1^T$ $C = U_2W_2V_2^T$, then $[B,C] = [U_1,U_2][W_1,W_2]([V_1,V_2]^T)$. In the study of so called MIMO channels in digital communication the Kronecker product has become very important. For instance, a Google search with the key words MIMO + "Kronecker product"+SVD will give you some idea of the applications of Kronecker products and SVD to the future generations of mobile phones.