University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 09, 14.11.2011

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. On the www-page there are program `hlp091.m` and `myf1d.m`. They were written in order to compute the line integral of a function $f(x,y) = c(1)*x^2 + c(2)*y^2 + c(3)*x*y + c(4)*x$ along the polygonal segment with vertices $(0,0),(2,0),(2,1)$. In this special case also the exact value of the integral was computed and compared to the numerical value.

(a) Run this program and study how it works. Use it to compute the line integral of $\sin(xy) + \exp(x-y)$ along the same path.

(b) Generalize this program to the three dimensional case.

2. Let a be an $n \times n$ matrix with singular values $s_1 \geq s_2 \geq \ldots \geq s_n$, and eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, $|\lambda_i| \geq |\lambda_{i+1}|$. Show that for all $k = 1, \ldots, n$, $|\lambda_1 \cdots \lambda_k| \leq s_1 \cdots s_k$.

3. The isoperimetric inequality says that if a curve of length L encloses a plane region with area A, then $A \leq \pi(L/(2*\pi))^2$ [this upper bound is the area of a disk with boundary of length L]. In other words, the isoperimetric ratio $4*\pi*A/L^2 \leq 1$.

(a) Familiarize yourself with the program myisoper.m on the www-page, which prompts the user to enter points in the plane and then plots a spline curve through the points and computes the isoperimetric ratio. Try to find examples of curves with as small isoperimetric ratio as possible.

(b) Simplify the program by excluding the use of splines and using polygons instead. For several sets of the same vertices, compute the ratios for both the spline and polygonal curves. What is your observation?

4. Consider the solution of the nonlinear system $f(x) = 0$ with

$$f_1(x) = x_1 + 3\log|x_1| - x_2^2, \quad f_2(x) = 2x_1^2 - x_1x_2 - 5x_1 + 1.$$

(a) Use numjaco.m to check that

$$J_f(x) = \begin{bmatrix} 1 + 3/x_1 & -2x_2 \\ 4x_1 - x_2 - 5 & -x_1 \end{bmatrix}.$$

FILE: ~/mme/demo11/d09/d09.tex — 21. lokakuuta 2011 (klo 17.29).

Do not mix numjaco with the built-in function numjac!

(b) Show that $x^{(0)} = (2,2)^T$ is not a good starting point for the usual Newton iteration.

(c) Show also that $x^{(0)} = (2,2)^T$ is a good starting point for the damped Newton iteration. In the damped Newton method the correction vector is divided by 2 until $|f(x_{new})| \le |f(x_{old})|$:

```
for j=1:nstep
%   h= myjac(x)\myf(x);       ANALYTIC
   % N.B. myjac takes a column vector x
   h= numjaco('myf',2,x',2)\myf(x);   %NUMERIC
   % N.B. numjaco takes a row vector x
   count=0;   % counts the number of dampings
   while ( (norm(myf(x-h)) >= norm(myf(x))))
     h=0.5*h;
     count=count+1;
   end
   x=x-h;
end
```

5. Jacobi's method for solving a linear $n \times n$-system $ax = b$ is based on the iteration

$$x_j^{(k+1)} = \left(b_j - \sum_{i=1, i \neq j}^{n} a_{ji} x_i^{(k)} \right) \Big/ a_{jj} \qquad (j = 1, \ldots, n)\,,$$

where the initial value $x^{(0)}$ is chosen arbitrarily. The iteration converges if a is diagonally dominant, i.e. if it is true that $w(a,j) > 0$ for all $j = 1, \ldots, n$, where

$$w(a,j) \equiv |a_{jj}| - \sum_{i=1, i \neq j}^{n} |a_{ji}|\,.$$

Write a MATLAB-program which generates a diagonally dominant linear system, solves it using the Jacobi method, and compares the result to the solution obtained by using the slash-operator $a\backslash b$. How does increasing the absolute values of the diagonal entries effect the accuracy of the results?