

University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 01, 12.9.2011

Problem sessions will be held on Monday at 16-18

N.B. The files mentioned in the exercises (if any) are available on the course homepage

<http://wiki.helsinki.fi/pages/viewpage.action?pageId=70225790>

1. Apply the recursion formula $x_0 = 1, x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}), n = 0, 1, 2, \dots$ for \sqrt{a} to compute $\sqrt{3}$. Print the results in the following format:

n	x(n)	Error
0	1	
.....		
6	...	

2. Approximations to the number π are given by the formula

$$p(n) = \sum_{k=0}^n \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).$$

Print the first few results in the same format as in problem 1.

3. In Solmu 2/2005 (<http://solmu.math.helsinki.fi/2005/2/>) the following problem was studied. Is it true that a continuous function $f : (0, \infty) \rightarrow (0, \infty)$ satisfying the conditions:

1. $f(2x) = 2f(x)$, and
2. $f(1) = c$

is always of the form $f(x) = cx$. In the article, the following counterexample was presented:

$$f(x) = 2^{-n}x^2 + 2^{n+1} \text{ for } x \in [2^n, 2^{n+1}),$$

where $n = 0, \pm 1, \pm 2, \dots$ Plot the graph of this function.

4. Let $(x_j, y_j), j = 0, 1, \dots, n$ be the vertices of a polygon with $(x_0, y_0) = (x_n, y_n)$. The area of the polygon is given by $a = \frac{1}{2} \sum_{i=1}^n t_i$ with $t_i = x_{i-1}y_i - x_iy_{i-1}$. Carry out the following steps for each of the regular polygons triangle, square and hexagon:

- Choose vertices and compute the area by school geometry.
- Compute the area by the formula and compare to the exact value.
- Plot the figure.

5. Hilbert's inequality says that for $a_k, b_k \geq 0$

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} \leq \pi \left(\sum_{m=0}^{\infty} a_m^2 \right)^{1/2} \left(\sum_{n=0}^{\infty} b_n^2 \right)^{1/2}.$$

Carry out a numerical verification of this inequality.

6. Consider a data set $(x_i, y_i), i = 1, \dots, n$. We define $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ and

$$\begin{aligned} ss_{xx} &= \sum (x_i - \bar{x})^2 &= \sum x^2 - n\bar{x}^2 \\ ss_{yy} &= \sum (y_i - \bar{y})^2 &= \sum y^2 - n\bar{y}^2 \\ ss_{xy} &= \sum (x_i - \bar{x})(y_i - \bar{y}) &= \sum xy - n\bar{x}\bar{y}. \end{aligned}$$

Write a MATLAB program that computes the correlation coefficient r of the data set, defined as

$$r^2 = \frac{ss_{xy}^2}{ss_{xx}ss_{yy}}.$$

Create a synthetic data $x_i = i * 0.1, y_i = 0.7 * x_i + c * error_i$ where $error$ is uniformly distributed in $(-0.1, 0.1)$ with mean 0. One expects that the correlation coefficient decreases when c increases from 0.5 to 1. Check this with MATLAB.