Introduction to MATLAB
HOMEWORK 2/SIR model for measles

In epidemiology, compartmental models are largely used to model infectious diseases in a population. Simulate the measles epidemic in a population.

Define the compartments S (for susceptible), I (for infectious) and R (for recovered). They are proportions of the population that are in the compartments. The (nonlinear) differential equation describing the epidemiology is (' is for the time derivative)

$$\begin{cases} S' = \mu - \lambda S - \mu S, \\ I' = \lambda S - \eta I - \mu I, \\ R' = \eta I - \mu R. \end{cases}$$

Here, $\mu = 1/75$ is the mortality rate, corresponding the 75 expected life years. In the first equation, the first μ corresponds to the birth rate. In this way, the population is stable,

$$(S + I + R)' = 0 \quad (check!),$$

if we set $(S + I + R)(0) = 1$. Parameter $\eta = 1/(8/360)$ is the recovery rate, and this corresponds to the average 8 days infectivity period of measles. The force of infection

$$\lambda = \lambda(I) = \beta I$$

describes how much infectious contacts there are in the population, and $\beta = 200$ describes how much potentially infectious contacts[1] there are overall (per year). To take into account the back-ground force of infection (from foreign populations), set

$$\lambda = \max(\lambda I, 10^{-6}).$$

Write a function for the epidemic differential equation. You can write the parameter values into the function as fixed parameters. Compute the solution at time interval [0,100]. You can use the initial values

$$S(0) = 0, \quad I(0) = 0, \quad R(0) = 1.$$

Plot the functions

$$t \mapsto S(t), \quad t \mapsto I(t), \quad t \mapsto R(t).$$

Note that the scale of I differs from the scales of S and R. Can you explain the periodicity of I?

[1] One can think also as follows: A susceptible makes potentially infectious contacts with rate β, and the probability that the contact person is infectious is I.