Introduction to LaTeX
Exercises 2 (Group 5)

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The `.tex` file containing your solutions to this exercise sheet should be emailed to `clifford.gilmore@helsinki.fi` before 15:00 on 22nd March.

1. Create a document titled, *LaTeX Solutions 2*, with you as the author.

2. Create a section called, *My First Table*, and in this section add the below table.

	Height (cm)	Age (years)
Ian	180	22
Michelle	172	31

3. Create a new section titled, *Real and Fourier Analysis*. Define your own theorem structure with the `newtheorem` command and then use it to state the below theorem.

   Let $K = \{(x, y) \in \mathbb{R} : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x\}$ and let $f : K \to \mathbb{R}$ be an integrable function. Then

$$\int_K f dm_2 = \int_{[0,1]} \left(\int_{[0,x]} f(x,y) dm_1(y) \right) dm_1(x) = \int_{[0,1]} \left(\int_{[y,1]} f(x,y) dm_1(x) \right) dm_1(y)$$

4. Using the `proof` environment accessed through the `amsthm` package, add a proof for the above theorem. (You don't need to give the real proof, any paragraph of text will do)

5. Add a subsection called *Fourier Analysis* to the section and reproduce the below text in your document. (Hint: create a *Question* structure with the `newtheorem` command and use the `eqnarray` environment to align the calculations)

Question 3.1. *Let $f(0) = |\theta|, \theta \in [-\pi, \pi]$. Prove that $\hat{f}(0) = \frac{\pi}{2}$ and*

$$\hat{f}(n) = \frac{-1 + (-1)^n}{\pi n^2}, \qquad n \neq 0.$$

Proof. It is easily seen that

$$\hat{f}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| \, \mathrm{d}\theta = \frac{1}{\pi} \int_0^{\pi} \theta \, \mathrm{d}\theta = \frac{\pi}{2}.$$

Next, for non-zero integers n we have,

$$\begin{aligned}
\hat{f}(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| e^{-in\theta} \, \mathrm{d}\theta \\
&= \frac{1}{\pi} \int_0^{\pi} \theta \cos n\theta \, \mathrm{d}\theta \\
&= \frac{1}{\pi} \left(\left[\frac{\theta \sin n\theta}{n} \right]_0^{\pi} - \frac{1}{n} \int_0^{\pi} \sin n\theta \, \mathrm{d}\theta \right) \\
&= -\frac{1}{n\pi} \left[-\frac{\cos n\theta}{n} \right]_0^{\pi} \\
&= \frac{\cos n\pi - 1}{\pi n^2} \\
&= \frac{(-1)^n - 1}{\pi n^2} = \begin{cases} 0 & \text{if } 2 \mid n, \\ -\frac{2}{\pi n^2} & \text{if } 2 \nmid n. \end{cases}
\end{aligned}$$

□