

$$\begin{array}{c}
 \forall x \frac{[\forall x (A \rightarrow B)]^3}{A \rightarrow B} \quad \forall x \frac{[\forall x A]^1}{A} \rightarrow E \quad \frac{[\forall x (B \rightarrow C)]^2}{B \rightarrow C} \forall E^* \\
 \hline
 B \qquad \qquad \qquad B \rightarrow C \\
 \hline
 C \qquad \qquad \qquad \rightarrow E \\
 \hline
 \qquad \qquad \qquad \forall I, **
 \end{array}$$

①

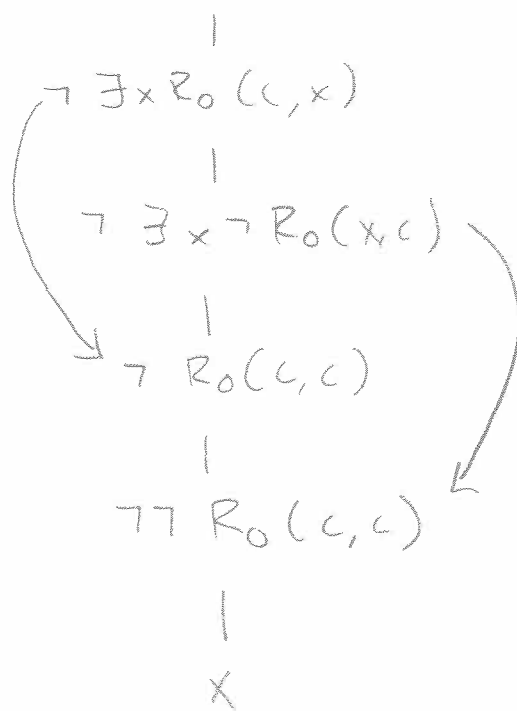
$$\begin{array}{c}
 \forall x C \qquad \qquad \qquad \rightarrow I, 1 \\
 \hline
 \forall x A \rightarrow \forall x C \qquad \qquad \qquad \rightarrow I, 2 \\
 \hline
 \forall x (B \rightarrow C) \rightarrow (\forall x A \rightarrow \forall x C) \qquad \qquad \qquad \rightarrow I, 3 \\
 \hline
 \forall x (A \rightarrow B) \rightarrow ((\forall x (B \rightarrow C)) \rightarrow (\forall x A \rightarrow \forall x C)) \qquad \qquad \qquad \rightarrow I, 3
 \end{array}$$

* , muuttuja vapaa itselleen

** , x ei vapaa hyökkäymättömissä oletuksissa

② Tutki lausetta saman tyylin avulla

$$\neg (\exists x R_0(c, x) \vee \exists x \neg R_0(x, c))^\vee$$



Lause on ristiriita, eli ei ole olemassa mallia joka sen toteuttaisi

③

$$\neg\neg (\forall x \forall y R_0(x, y) \wedge \neg \exists x \forall y R_0(x, y)) \checkmark$$

$$\forall x \forall y R_0(x, y) \wedge \neg \exists x \forall y R_0(x, y) \checkmark$$

$$\forall x \forall y R_0(x, y)$$

$$\neg \exists x \forall y R_0(x, y)$$

$$\neg \forall y R_0(c_0, y) \checkmark$$

$$\neg R_0(c_0, c_1)$$

$$\forall y R_0(c_0, y)$$

$$R_0(c_0, c_1)$$

X

④

$$\exists x_0 \exists x_1 \neg ((P_0(x_0) \leftrightarrow P_0(x_1))) \checkmark$$

$$\downarrow$$
$$\exists x_1 \neg ((P_0(c_0) \leftrightarrow P_0(x_1)))$$

$$\downarrow$$
$$\neg ((P_0(c_0) \leftrightarrow P_0(c_1)))$$

$$\begin{array}{cc} / & \backslash \\ P_0(c_0) & \neg P_0(c_0) \\ | & | \\ \neg P_0(c_1) & P_0(c_1) \end{array}$$

Konstruimme mallin:

$$M = \{ \{0, 1\}, P_0, c_0, c_1 \}$$

missä

$$P_0^M = \{0\}, c_0^M = 0, c_1^M = 1$$