Dependence logic

Problems 1

Tuesday 22.3.2011

1. For a first-order formula ϕ, let us consider the operations $\phi \mapsto \phi^p$ and $\phi \mapsto \phi^d$. Show that ϕ^p and ϕ^d are always in negation normal form. Show also that $(\phi^d)^d = (\phi^p)^p = \phi^p$.

2. Show that ϕ^p is logically equivalent to ϕ and ϕ^d to $\neg\phi$.

3. Let $M = \{0, 1, 2\}$. Consider the following team X of M with domain $\{x_0, x_1, x_2\}$:

	x_0	x_1	x_2
s_0	1	2	2
s_1	2	1	2
s_2	0	1	2

Is X of type ϕ (that is, does $M \models_X \phi$ hold) if:

1. $\phi := x_0 = x_2$ or $\phi := \neg x_0 = x_2$
2. $\phi := \exists x_0 (x_0 = x_2)$
3. $\phi := \forall x_3 =\!(x_2)$
4. $\phi := (=\!(x_0, x_1) \vee =\!(x_1, x_2))$

4. Let $\mathcal{M} = (M, R^{\mathcal{M}})$ where $R^{\mathcal{M}} \subseteq M^3$ and $\phi := \forall x_0 \forall x_1 \exists x_2 R(x_0, x_1, x_2)$. Show that $\mathcal{M} \models_{\{\emptyset\}} \phi$ if and only if ϕ is true in $\mathcal{M}$ as a sentence of first-order logic. Let $\psi := \forall x_0 \forall x_1 \exists x_2 (=\!(x_0, x_2) \wedge R(x_0, x_1, x_2))$. Can you find a first-order sentence equivalent to ψ?

5. Show that $\models \forall x_0 \forall x_1 (x_1 = c \rightarrow =\!(x_0, x_1))$.

6. Let $\mathcal{M} = (\mathbb{N}, +, \times, 0, 1)$. Which teams X of $\mathcal{M}$ with domain $\{x_0, x_1\}$ are of type

1. $=\!(x_0, x_0 + x_1)$
2. $=\!(x_0 \times x_0, x_1 \times x_1)$