

Intermezzo

Consider the EBR



Corresponding ODE for A?

$$\frac{dA}{dt} = -2kA^2 \quad \text{or} \quad \frac{dA}{dt} = -kA^2$$

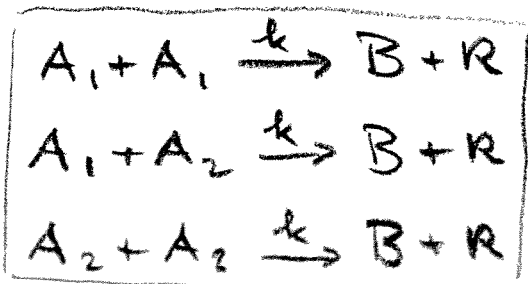
Write

(i)
$$\frac{dA}{dt} = -cA^2$$

for some $c > 0$ we still have to decide on.

Thought experiment:

Split A-pop. arbitrarily into two groups labelled A_1 and A_2 , which remain chemically equivalent.



Then

(ii)

$$\begin{aligned} \frac{dA_1}{dt} &= -cA_1^2 - kA_1A_2 \\ \frac{dA_2}{dt} &= -kA_1A_2 - cA_2^2 \end{aligned}$$

Since $A = A_1 + A_2$ (concentrations)
we have from (i)

(iii)
$$\frac{d(A_1 + A_2)}{dt} = -c(A_1 + A_2)^2 =$$

$$= -cA_1^2 - 2cA_1A_2 - cA_2^2$$

and from (ii)

(iv)
$$\frac{d(A_1 + A_2)}{dt} = -cA_1^2 - 2kA_1A_2 - cA_2^2$$

Since (iii) and (iv) must be equal for any subdivision A_1, A_2 , it follows that $c = k$.

The correct ODEs are therefore:

$$\frac{dA}{dt} = -kA^2$$

$$\frac{dB}{dt} = \frac{1}{2}kA^2$$

$$\frac{dR}{dt} = \frac{1}{2}kA^2$$

(Some for every two A-moles that disappears only one B and one R mole appears. Hence the factor $\frac{1}{2}$.)

If B happens to be the same as A:

$$\frac{dA}{dt} = -kA^2 + \frac{1}{2}kA^2 = -\frac{1}{2}kA^2$$

$$\frac{dR}{dt} = \frac{1}{2}kA^2$$

(end of intermezzo)